Lecture 17

Kinematics and Motion Capture

Interactive Computer Graphics
Stanford CS248, Winter 2021
Today

- KINEMATICS: we are going to describe how objects move, without considering the underlying forces that generate that motion
Forward kinematics

Articulated skeleton
- Topology (what’s connected to what)
- Geometric relations from joints
- Tree structure (in absence of loops)

Joint types
- Pin (1D rotation)
- Ball (2D rotation)
- Prismatic joint (translation)
Forward kinematics

Example: simple two segment arm in 2D

Object space position of part

Warning: Z-up Coordinate System
Forward kinematics

Animator provides angles, and computer determines position \( p \) of end-effector

To transform point \( p \) with object space representation \((0, l_2)\) into world space:

- **Rotate clockwise by** \( \theta_2 \)
- **Translate by** \((0, l_1)\)
- **Rotate clockwise by** \( \theta_1 \)

\[
\begin{align*}
p_z &= l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \\
p_x &= l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)
\end{align*}
\]

Warning: Z-up Coordinate System
Forward kinematics

Animation is described as angle parameter values as a function of time: $\theta_1(t), \theta_2(t)$

$$p_z = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

$$p_x = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$

Warning: Z-up Coordinate System
Example: walk cycle

Articulated leg:

- Hip
  - Upper leg
    - Knee
      - Lower leg
        - Ankle
          - Foot
        - Foot (ankle rot)
    - Upper leg (hip rot)
      - Hip rotate
        - Lower leg (knee rot)
          - Hip rotate + knee rot
            - Foot (ankle rot)

Watt & Watt

Slide credit: Tom Funkhouser, Ren Ng
Example: walk cycle

Hip joint angle
Example: walk cycle

Knee joint angle

Watt & Watt

Slide credit: Tom Funkhouser, Ren Ng
Example: walk cycle

Ankle joint angle

Watt & Watt
Example: walk cycle
Skinning: how to transform surface mesh vertices according to skeleton transforms

Skeleton joint transforms: $T_1, T_2$

Image credit: Ladislav Kavan
Vertex $i$ on mesh

Image credit: Ladislav Kavan
Rigid body skinning

- One idea: transform mesh vertices according to transform for nearby skeleton joint

Original pose

Blue verts = associated with first joint
Red verts = associated with second joint

Vertices transforms according to corresponding joint transform (notice surface interpenetration)
Linear blend skinning *

Mesh vertices transformed by *linear combination* of nearby joint transforms

Very common technique for character animation in games

\[ v'_i = \sum_{j=1}^{N} w_{ij} T_j v_i \]

\[ = \left( \sum_{j=1}^{N} w_{ij} T_j \right) v_i \]

Here:

- \( v_i \) = rest object space vertex position
- \( T_j \) = transform for bone \( j \)
- \( w_{ij} \) = weight of bone \( j \) on vertex \( i \)
- \( N \) = number of bones

* Also called “matrix palette skinning” or “skeletal subspace deformation” (SSD)

Image credit: Ladislav Kavan
Linear blend skinning

- Transform mesh vertices according to linear combination of transforms for nearby skeleton joint

Original pose

After transform

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Shortcomings of linear blend skinning

- Loss of volume under large transformations

Many more advanced solutions in literature: dual-quaternion skinning, joint-based deformers, etc.
Skinning example

Courtesy Matthew Lailler via Keenan Crane via Ren Ng
Rigging

- “Rigging” is the process of attaching a set of animation controls to a mesh
- In the case of linear blend skinning: it is attaching a skeleton to the mesh (and setting per vertex blend weights)
- In the image to the right, the brightness of the rendering visualizes the influence of the selected joint on the mesh vertices.

Image credit: Unreal Engine 4 Documentation (see “Paint Skin Weights” Tool)
Different ways to obtain joint angles

- Hand animate values (as discussed above)
  - For example, by defining splines that give angle over time

- Measure angles from a performance via motion capture

- Solve for angles based on higher-level goal (optimization)
Motion Capture
Motion capture

- Data-driven approach to creating animation sequences
  - Record real-world performances (e.g. person executing an activity)
  - Extract pose as a function of time from the data collected

Motion capture room for ShaqFu
Optical motion capture

Source: http://fightland.vice.com/blog/ronda-rousey-20-the-queen-of-all-media

Ronda Rousey in Electronic Arts’ motion capture studio
Optical motion capture

- Affix markers to joints of subject
- Compute 3D positions by triangulation from multiple cameras
- 8+ cameras, 240 Hz, occlusions are difficult

Slide credit: Steve Marschner
Motion capture pros and cons

- **Strengths**
  - Can capture large amounts of real motion data quickly
  - Realism can be high

- **Weaknesses**
  - Complex and costly set-ups (but progress in computer vision is changing this)
  - Occlusions (e.g., hard to capture ballroom dance)
  - Captured animation may not meet artistic needs, requiring alterations
Challenges of facial animation

- "Uncanny valley"
  - In robotics and graphics
  - As artificial character appearance approaches human realism, our emotional response goes negative, (until appearance achieves a sufficiently convincing level of realism in expression)
Challenges of facial motion capture

Final Fantasy Spirits Within (2001)
Facial motion capture

Discovery, “Avatar: Motion Capture Mirrors Emotions”, https://youtu.be/1wK1lxr-UmM
Aside: lower-cost forms of capture
Microsoft XBox 360 Kinect

Illuminant (Infrared Laser + diffuser)

RGB CMOS Sensor 640x480 (w/ Bayer mosaic)

Monochrome Infrared CMOS Sensor (Aptina MT9M001) 1280x1024 **

** Kinect returns 640x480 disparity image, suspect sensor is configured for 2x2 pixel binning down to 640x512, then crop
Infrared image of Kinect illuminant output
Infrared image of Kinect illuminant output

Credit: www.futurepicture.org
Depth from “disparity” using structured light

System: one light source emitting known beam + one camera measuring scene appearance
If the scene is at reference plane, image that will be recorded by camera is known
Movement of observed dot from reference gives depth.

\[
\frac{z}{b} = \frac{f}{x + d}
\]

\[
z = \frac{bf}{x + d}
\]

Reference plane

Single spot illuminant is inefficient!
(Must “scan” scene to get depth, so high latency to retrieve a single depth image. Hence the dot pattern on the Kinect)
Xbox One Sensor

- Time-of-flight sensor (not based on structured light like the original Kinect)
- Measure phase offset of light reflected off environment
  - Phase shift proportional to distance from object
- “Computer vision” challenges in obtaining high-quality signal:
  - Flying pixels
  - Segmentation
  - Motion blur

Another TOF camera:
Creative Depth Camera

Image credit: V. Castaneda and N. Navab
Extracting the player’s 2D skeleton
(permitting full-body game input)

Challenge: how to determine player’s position and motion from (noisy) depth images... without consuming a large fraction of the XBox 360’s compute capability?

[Shotton et al. 2011]
Key idea: classify pixels into body regions

Shotton et al. represents body with 31 regions
Pixel classification

For each pixel: compute features from depth image

\[ f_\theta(I, X) = d_1\left(\frac{X + u}{d_1(X)}\right) + d_1\left(\frac{X + v}{d_1(X)}\right) \]

Where \( \theta = (u, v) \) and \( d_1(X) \) is the depth image value at pixel \( X \).

Features are cheap to compute + can be computed for all pixels in parallel
- Features do not depend on velocities: only information from current frame

Classify pixels into body parts using randomized decision forest classifier
- Trained on 100K motion capture poses + database of rendered images as ground truth

Result of classification: 

\[ P(c | I, x) \]

(probability pixel \( x \) in depth image \( I \) is body part \( c \))

Per-pixel probabilities pooled to compute 3D spatial density function for each body part \( c \).

(joint angles inferred from this density)
Modern computer vision approaches

- 2D (but not 3D) skeleton from single RGB image

Ongoing research to obtain high-quality 3D poses

Image credits: Cao et al 2017, Simon et al 2017
Single camera facial performance capture

Input video frame

DNN
(trained on “ground truth” mesh data output by an expensive video processing pipeline that used 9 video cameras)

Output 3D mesh

[Image credit: “Production-Level Facial Performance Capture Using Deep Convolutional Neural Networks”, Lehtinen et al 2017]
Single smartphone camera facial performance capture (Apple Animoji)
So far... we’ve discussed hand animating or directly measuring joint positions

Inverse Kinematics
(computer solves for joint angles based on high-level goal)
Example: inverse kinematics
Example: inverse kinematics

Example 12: IK-driven robot claw
Inverse kinematics

Input: animator provides position of end-effector
Output: computer must determine joint angles that satisfy constraints
Inverse kinematics

Direct inverse kinematics: for two-segment arm, can solve for parameters analytically (not true for general N-link problem)

\[ \theta_2 = \cos^{-1} \left( \frac{p_2^2 + p_3^2 - l_1^2 - l_2^2}{2l_1l_2} \right) \]

\[ \theta_1 = \frac{-p_zl_2 \sin(\theta_2) + p_x(l_1 + l_2 \cos(\theta_2))}{p xl_2 \sin(\theta_2) + p_z(l_1 + l_2 \cos(\theta_2))} \]
Inverse kinematics

- Why is the problem hard?
  - Multiple solutions in configuration space (and these may not be nearby, causing jumps from frame-to-frame)
  - Solution may not be possible
Inverse kinematics

- Numerical solution to general N-link IK problem
  - Choose an initial configuration
  - Define an error metric (e.g. square of distance between goal and end effector’s current position)
  - Apply *optimization method* to solve for joint angles given the desired (goal) end effector position
A few bits on optimization
(a commonly used tool in graphics)
Optimization problem in standard form

- Can formulate most continuous optimization problems this way:

  "objective": how much does solution \( x \) cost?

  $$\min_{x \in \mathbb{R}^n} f_0(x)$$

  subject to

  $$f_i(x) \leq b_i, \quad i = 1, \ldots, m$$

  "constraints": what must be true about \( x \)? ("\( x \) is feasible")

- **Optimal solution** \( x^* \) has smallest value of \( f_0 \) among all feasible \( x \)

- Q: What if we want to *maximize* something instead?

  A: Just flip the sign of the objective!

- Q: What if we want *equality* constraints, rather than inequalities?

  A: Include two constraints: \( g(x) \leq c \) and \(-g(x) \leq -c\)
Local vs. global minima

- *Global* minimum is absolute best among all possibilities
- *Local* minimum is best “among immediate neighbors”

Philosophical question: does a local minimum “solve” the problem?
Optimization problem, visualized

\[
\begin{align*}
\min_{x \in \mathbb{R}^2} & \quad x_1^2 - x_2^2 \\
\text{s.t.} & \quad x_1^2 + x_2^2 - 1 \leq 0
\end{align*}
\]

Q: Is this an optimization problem in standard form?  
A: Yes

Q: Where is the optimal solution?  
A: There are two, (0,1), (0,-1)
Existence and uniqueness of minimizers

- Already saw that (global) minimizer is not unique
- Does it always exist? Why?
- Just consider all possibilities and take the smallest one, right?

\[ f_0(x) \]

\[ \min_x x \quad x \in \mathbb{R} \]

- **WRONG!** Not all objectives are bounded from below.
Feasibility

- Ok, but suppose the objective is bounded from below
- Then we can just take the best feasible solution, right?

value of objective doesn’t depend on \( x \);
all feasible solutions are equally good

\[
\begin{align*}
\min_{x \in \mathbb{R}^n} & \quad 0 \\
\text{subject to} \quad & f_i(x) \leq b_i, \ i = 1, \ldots, m
\end{align*}
\]

- Not if there aren’t any!
- Not all problems have solutions!
Q: Is this problem feasible?

A: No—the two sublevel sets (points where \( f_i(x) \leq 0 \)) have no common points, i.e., they do not overlap.

\[
\begin{align*}
\min_{x \in \mathbb{R}^2} & \quad \sin(x_1) + x_2^2 \\
\text{s.t.} & \quad (x_1 - 2)^2 + x_2^2 \leq 1, \\
& \quad x_1 \leq -1
\end{align*}
\]
Existence and uniqueness of minimizers, cont.

- Even being bounded from below is not enough:

\[ f(x) \]

\[
\min_{x \in \mathbb{R}} e^{-x}
\]

- No matter how big \( x \) is, we never achieve the lower bound (0)
Characterization of minimizers

- Ok, so we have some sense of when a minimizer might exist.
- But how do we know a given point $x$ is a minimizer?

- Checking if a point is a global minimizer is (generally) hard.
- But we can certainly test if a point is a local minimum (ideas?).
- (Note: a global minimum is also a local minimum!)
Characterization of local minima

- Consider an objective $f_0: \mathbb{R} \to \mathbb{R}$. How do you find a minimum?
- (Hint: you may have memorized this formula in high school!)

$$f_0'(x^*) = 0$$

...but what about this point?

Also need to check second derivative (how?)
Make sure it’s positive
But what does this all mean for more general functions $f_0$?
Optimality conditions (unconstrained)

- In general, our objective is $f_0 : \mathbb{R}^n \rightarrow \mathbb{R}$
- How do we test for a local minimum?
- 1st derivative becomes *gradient*; 2nd derivative becomes *Hessian*

$$\nabla f := \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

**GRADIENT**
(measures "slope")

$$\nabla^2 f := \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial f}{\partial x_n^2} \end{bmatrix}$$

**HESSIAN**
(measures "curvature")

- Optimality conditions?

$$\nabla f_0 (x^*) = 0$$

1st order

$$\nabla^2 f_0 (x^*) \succeq 0$$

2nd order

*positive semidefinite (PSD)*
($u^T A u \geq 0$ for all $u$)
Convex optimization

- Special class of problems that are almost always “easy” to solve (polynomial-time!)
- Problem is convex if it has a convex domain and convex objective

Why care about convex problems in graphics?
- can make guarantees about solution (always the best)
- doesn’t depend on initialization (strong convexity)
- often efficient to solve
Sadly, life is not usually that easy. How do we solve optimization problems in general?
Descent methods

An idea as old as the hills:
Gradient descent (1D)

- Basic idea: follow the gradient “downhill” until it’s zero
- (Zero gradient was our 1st-order optimality condition)

\[ \frac{d}{dt} x(t) = -f'_0(x(t)) \]

- Do we always end up at a (global) minimum?
- How do we compute gradient descent in practice?
Gradient descent algorithm (1D)

- “Walk downhill”

\[ x_{k+1} = x_k - \tau f'_0(x_k) \]

- Q: How do we pick the step size?
- If we’re not careful, we’ll go zipping all over the place; won’t make any progress.

- Basic idea: use “step control” to determine step size based on value of objective and derivatives
- For now we will do something simple: make \( \tau \) small!
Gradient descent algorithm (n-D)

Q: How do we write gradient descent equation in general?

\[ \frac{d}{dt} x(t) = -\nabla f_0(x(t)) \]

Q: What’s the corresponding discrete update?

\[ x_{k+1} = x_k - \tau \nabla f_0(x_k) \]

Basic challenge in nD:
- solution can “oscillate”
- takes many, many small steps
- very slow to converge
Simple inverse kinematics algorithm

- **What is the objective?**
  - Distance from end effector position (given current joint parameters) to target position.

  \[ f_0(\theta) = \| p_{current} - p_{target} \|^2 \]

- **Constraints?**
  - Could limit range of motion of a joint

- **How to optimize for joint angles:**
  - Compute gradient of objective with respect to joint angles
  - Apply gradient descent
Many uses of optimization in animation (and graphics in general)

Sumit Jain, Yuting Ye, and C. Karen Liu, “Optimization-based Interactive Motion Synthesis”
Summary

- Kinematics: how objects move, without regard to forces that create this movement

- Today: multiple ways of obtaining joint motion
  - Direct hand authoring of joint angles
  - Via measurement (motion capture)
  - As a result of solving for angles that yield a particular higher level result (inverse kinematics)

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