Lecture 14:

Color

Interactive Computer Graphics
Stanford CS248, Winter 2020

Tunes

Keb Mo
“Love Blues”
(Keb Mo)

“Love [0,0,1] in sRGB color space.”
- Keb Mo

Cannon Beach, Oregon

Rio de Janeiro, Brazil

Zhangye Danxia Geological Park, China

Credit: http://parade.com/63549/linzlowe/where-in-the-world-are-these-incredible-rainbow-mountains

Meow Wolf, Santa Fe, NM

Vietnam

Sydney Harbor, Australia

Great Barrier Reef


Starry Night, Van Gogh

Marilyn Monroe, Andy Warhol

Why do we need to be able to talk precisely about color?


"An easy choice"
WIN
CONIA
SONY
LG

What is color?

- Color is a phenomenon of human perception; it is not a universal property of light

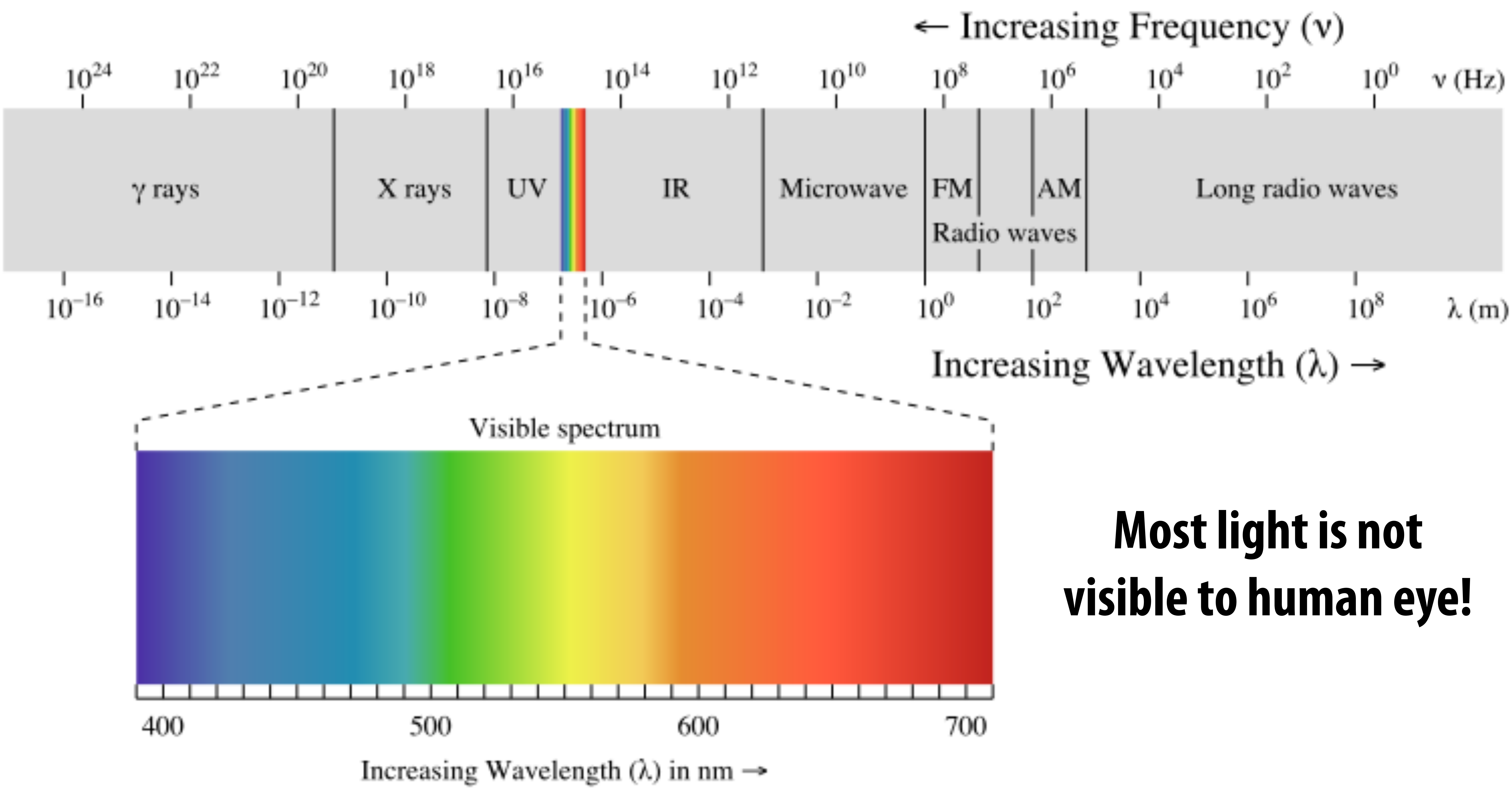
- Colors are the perceptual sensations that arise from seeing light of different spectral power distributions

- Technically speaking, different wavelengths of light are not "colors"

The Physical Basis of Color

(Review: what is light?)

Light is electromagnetic radiation (oscillating electromagnetic field)
Perceived color is *related to* frequency of oscillation



Most light is not visible to human eye!

Image credit: Licensed under CC BY-SA 3.0 via Commons
https://commons.wikimedia.org/wiki/File:EM_spectrum.svg#/media/File:EM_spectrum.svg

Spectral power distribution (SPD)

- **Salient property in measuring light**
  - **The amount of light present at each wavelength**
  - **Units:**
    - **Radiometric units / nanometer (e.g. watts / nm)**
    - **Can also be unit-less**
  - **Often use "relative units" scaled to maximum wavelength for comparison across wavelengths when absolute units are not important (the diagrams in this lecture do this)**

Spectral power distribution of common light sources

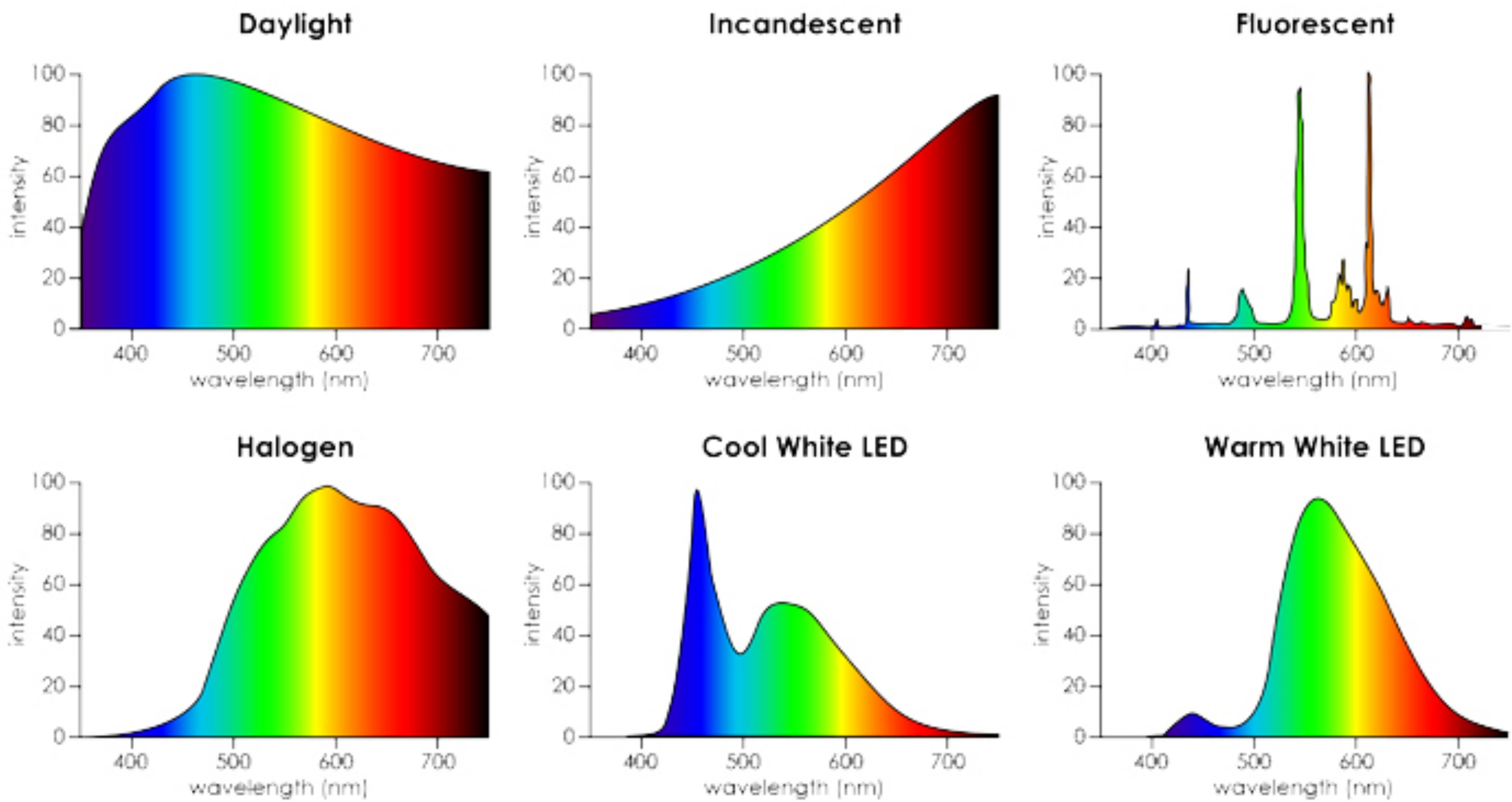
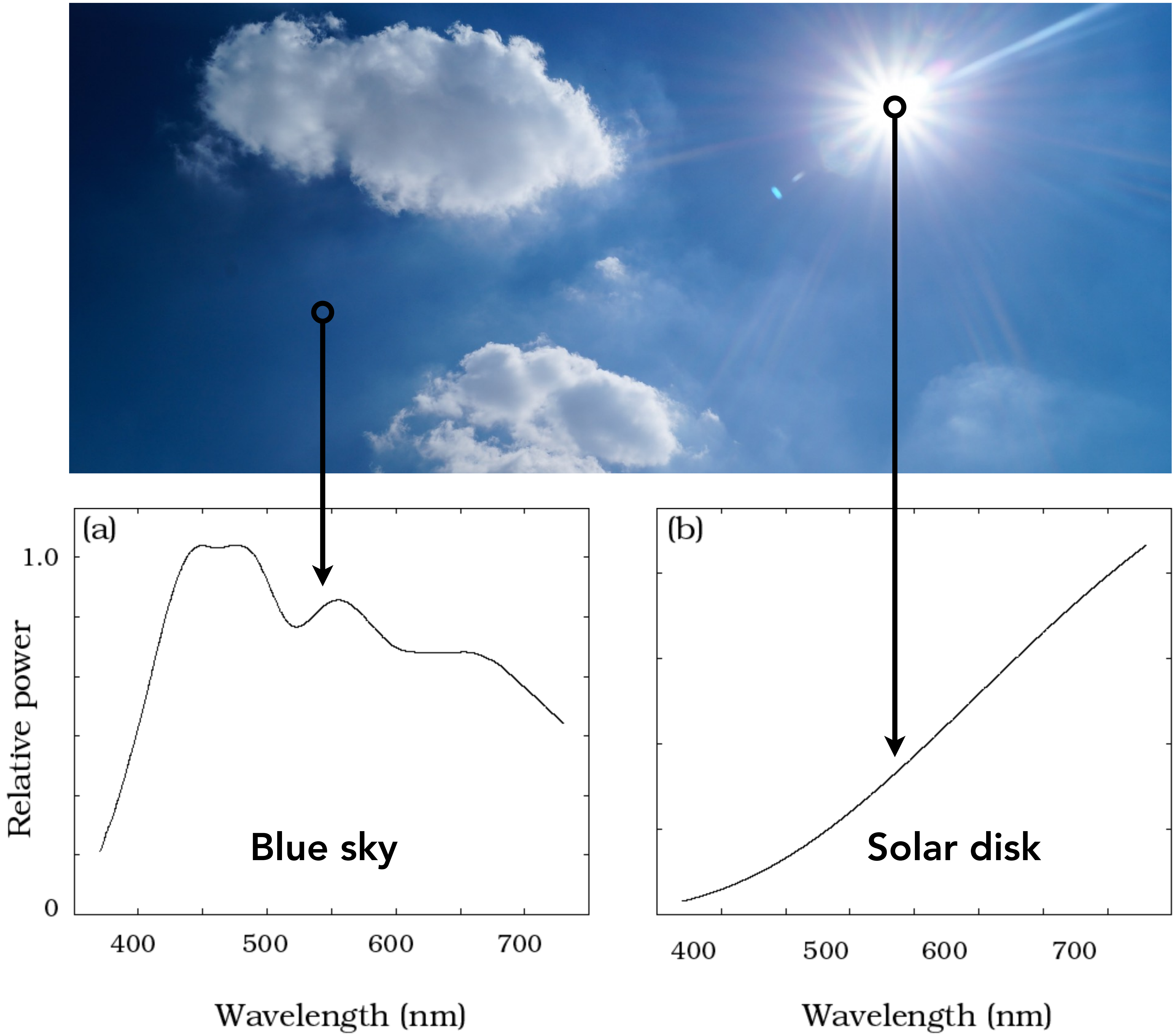


Figure credit:

admesy
ADVANCED MEASUREMENT SYSTEMS

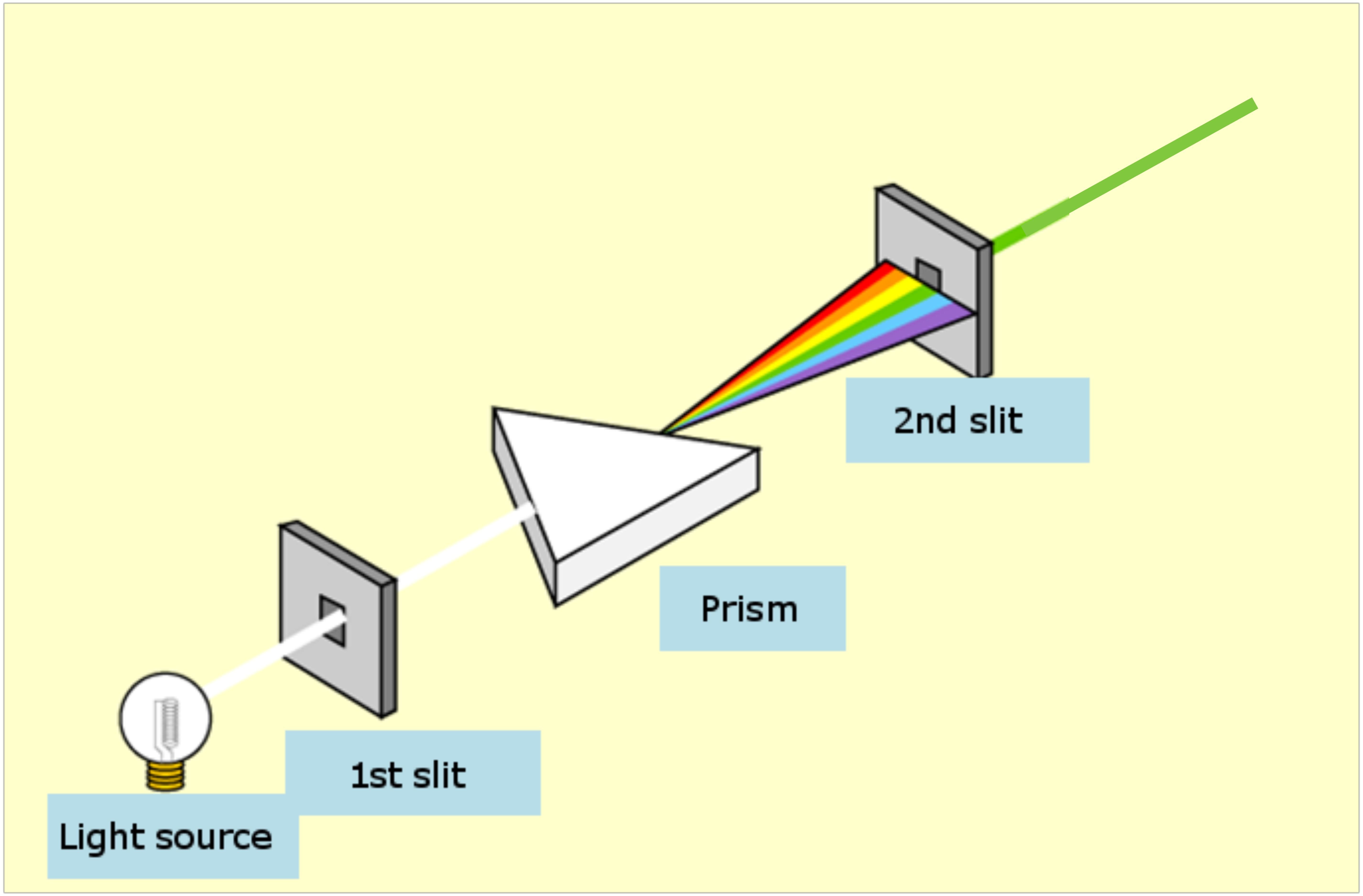
Daylight spectral power distributions vary



[Brian Wandell]

credit: Science Media Group.

Monochromator



A monochromator delivers light of a single wavelength from a light source with broad spectrum.
Control which wavelength by angle of prism.

Hegyi et al. "Introduction to Practical Biochemistry"
http://elte.prompt.hu/sites/default/files/tananyagok/IntroductionToPracticalBiochemistry/ch04s07.html

Superposition (linearity) of spectral power distributions

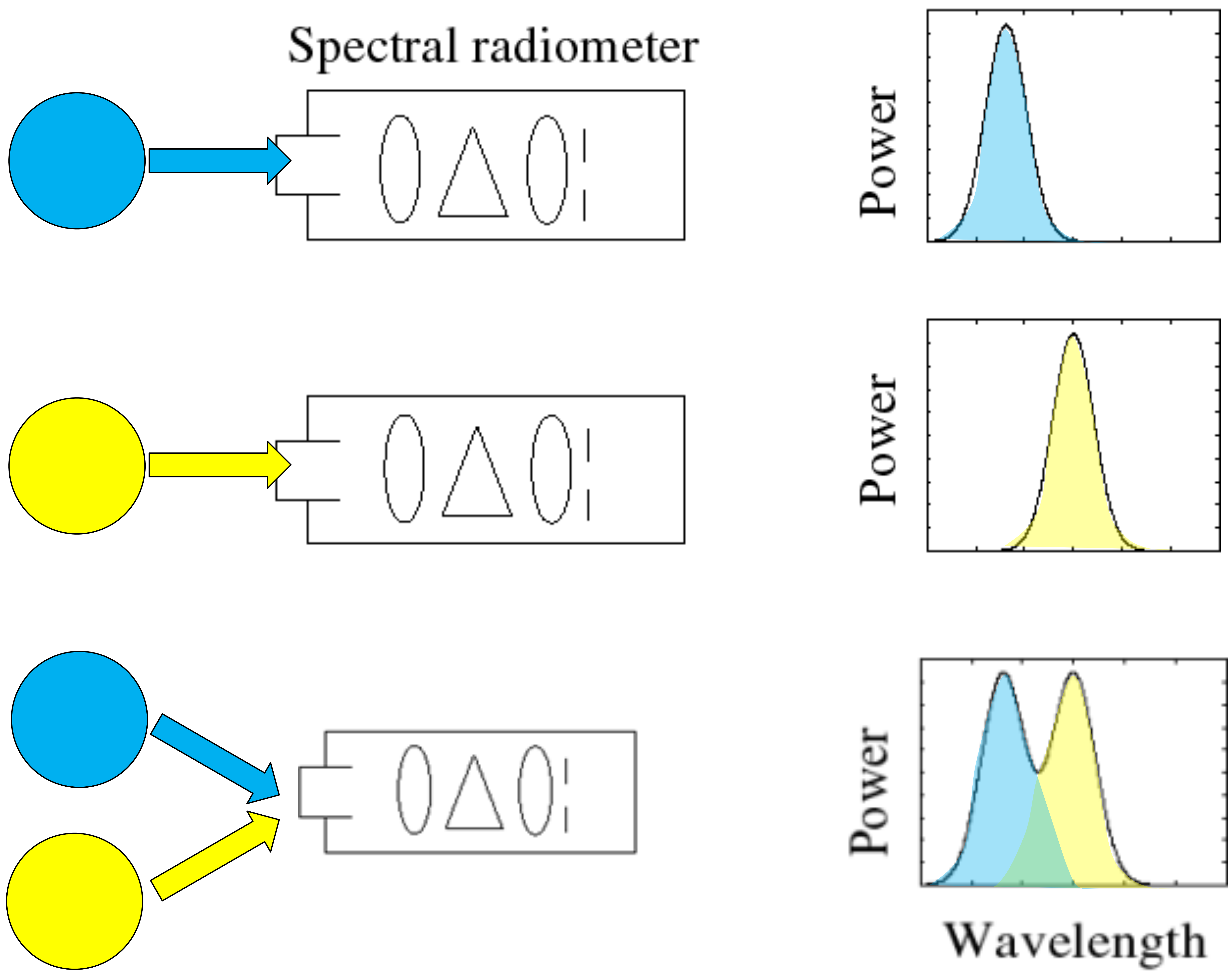
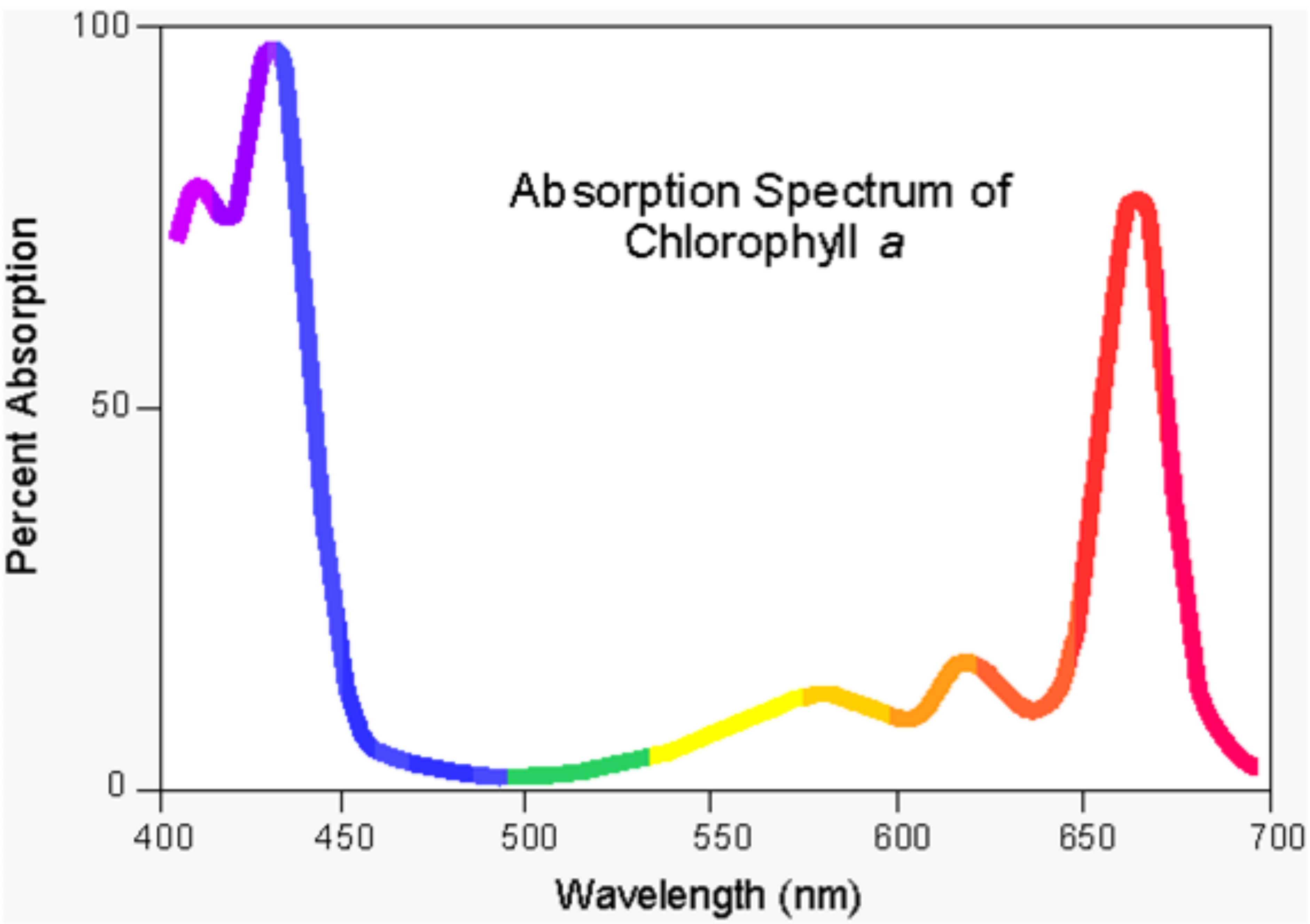


Figure credit: Brian Wandell

Absorption spectrum

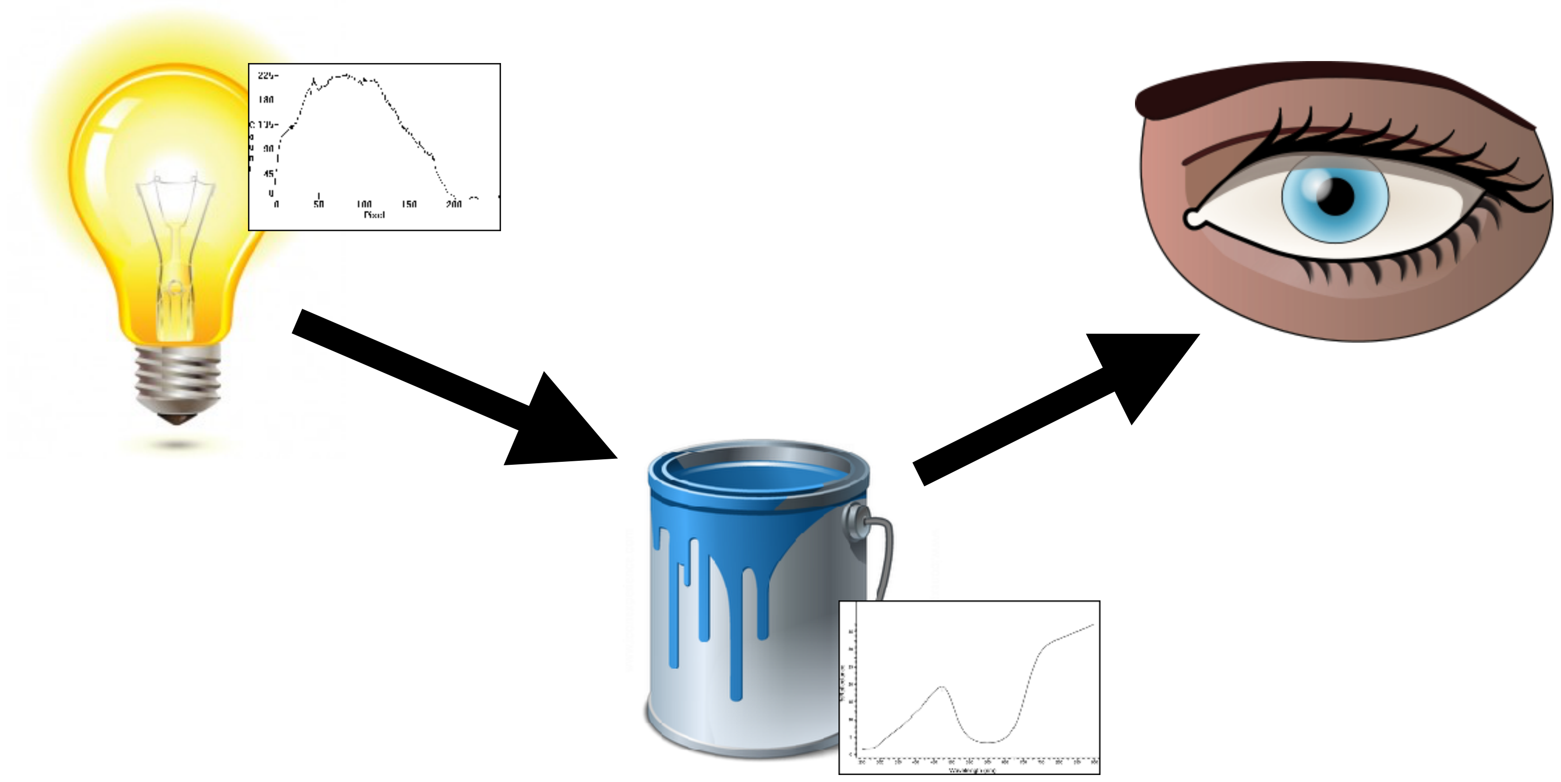
- Emission spectrum is *intensity* as a function of frequency
- Absorption spectrum is *fraction absorbed* as function of frequency



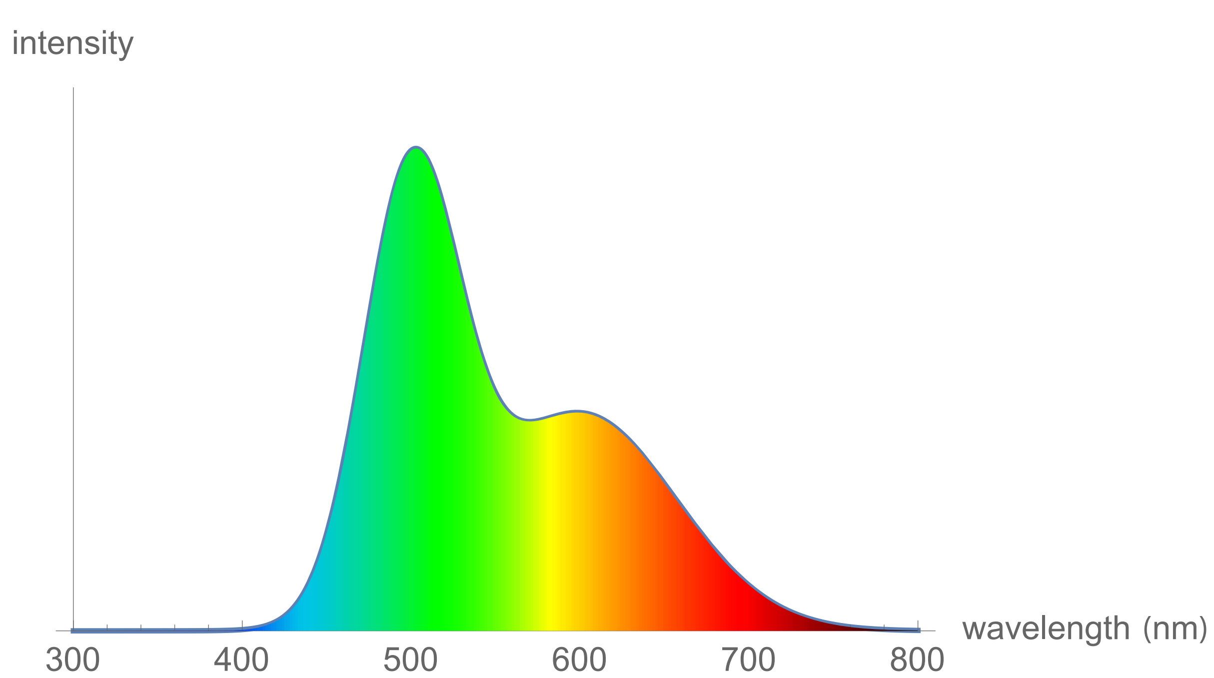
Q: What does an object with this absorption spectrum look like?

Interaction of emission and reflection

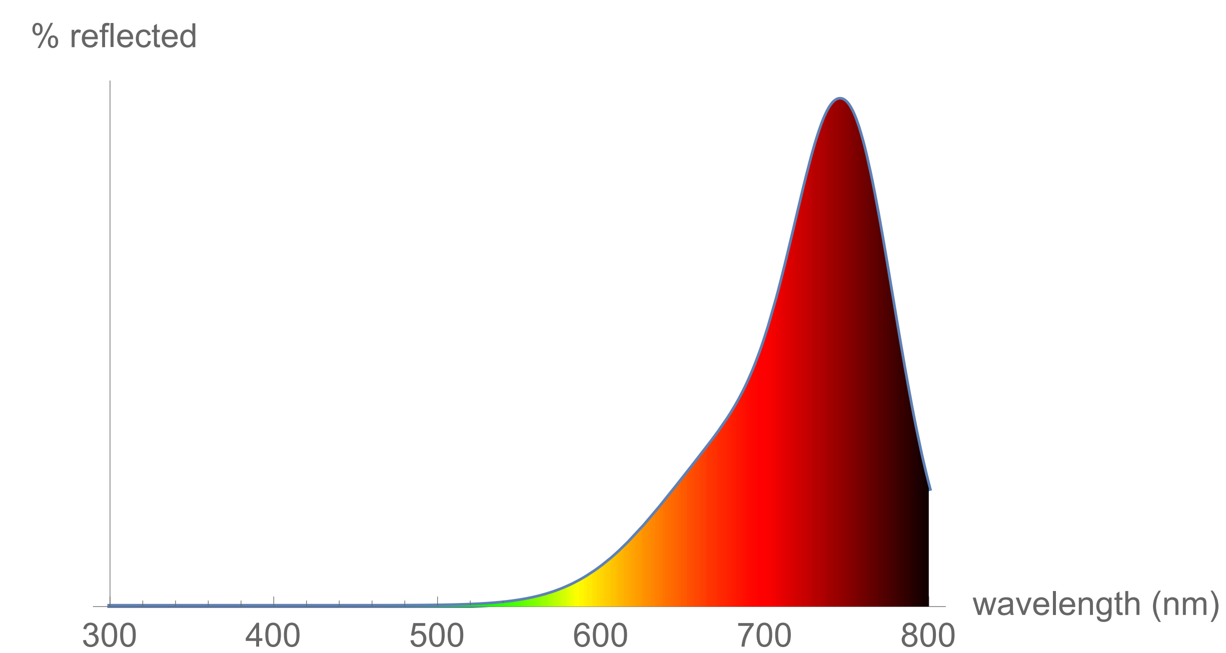
- Consider what happens when light gets reflected from a surface
  - ν — frequency (Greek "nu")
  - Light source has emission spectrum f(ν)
  - Surface has reflection spectrum g(ν)
  - Resulting intensity is the *product* f(ν)g(ν)



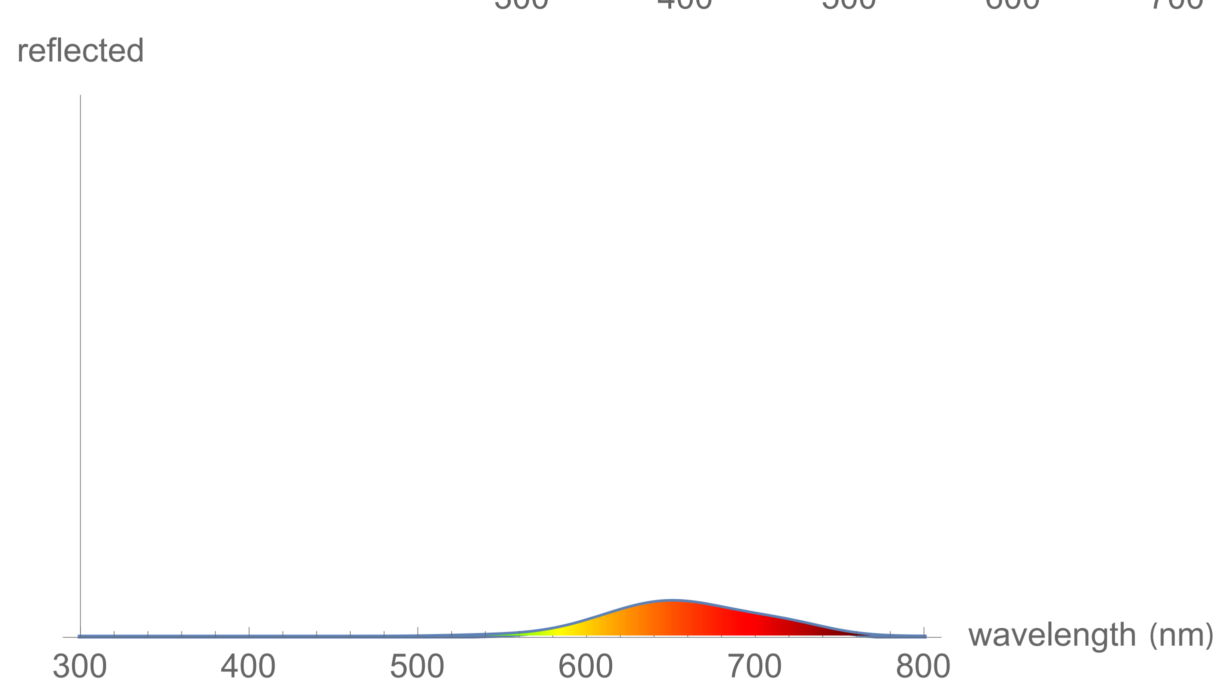
f(ν)



g(ν)



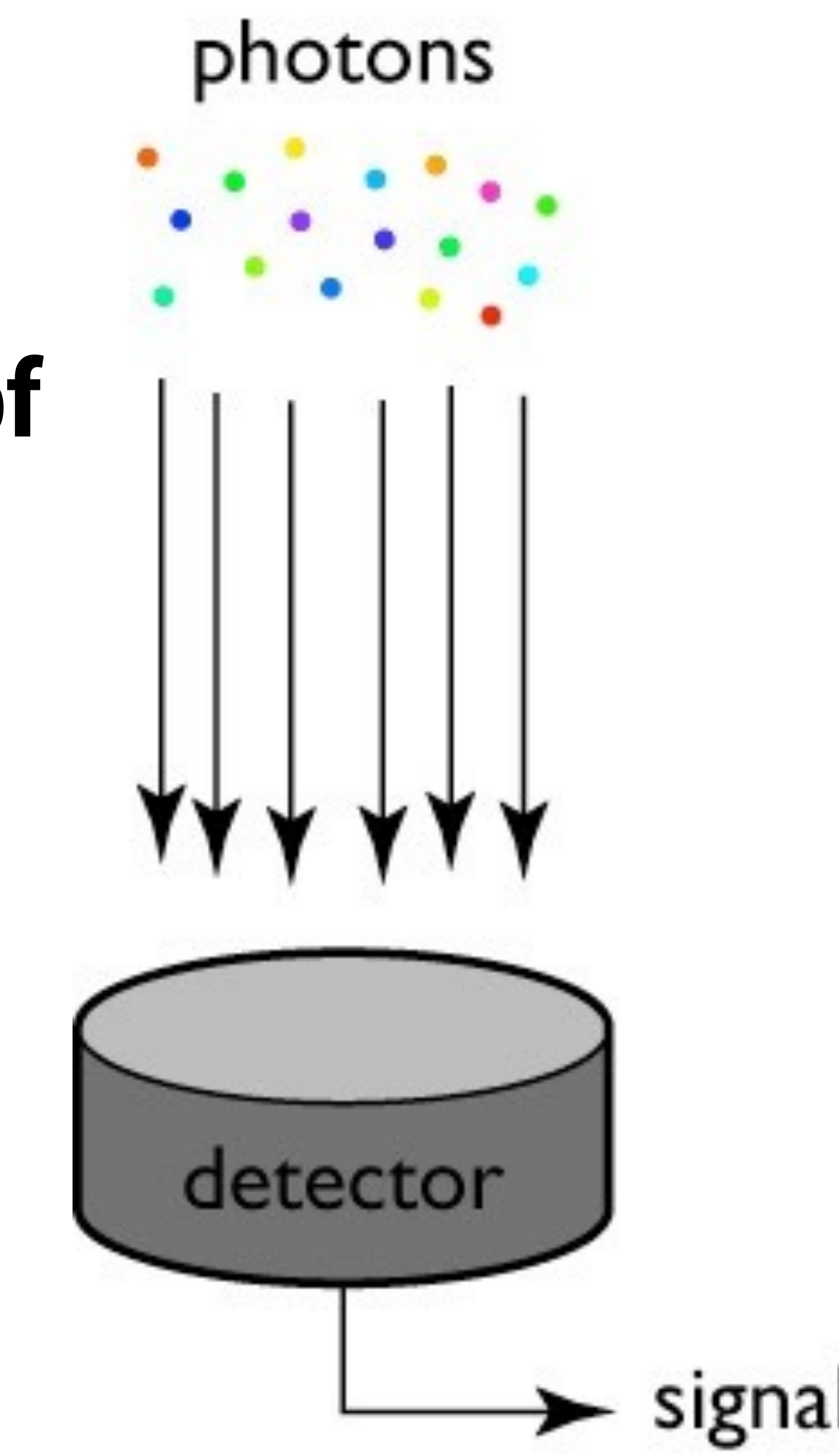
f(ν)g(ν)



Measuring Light

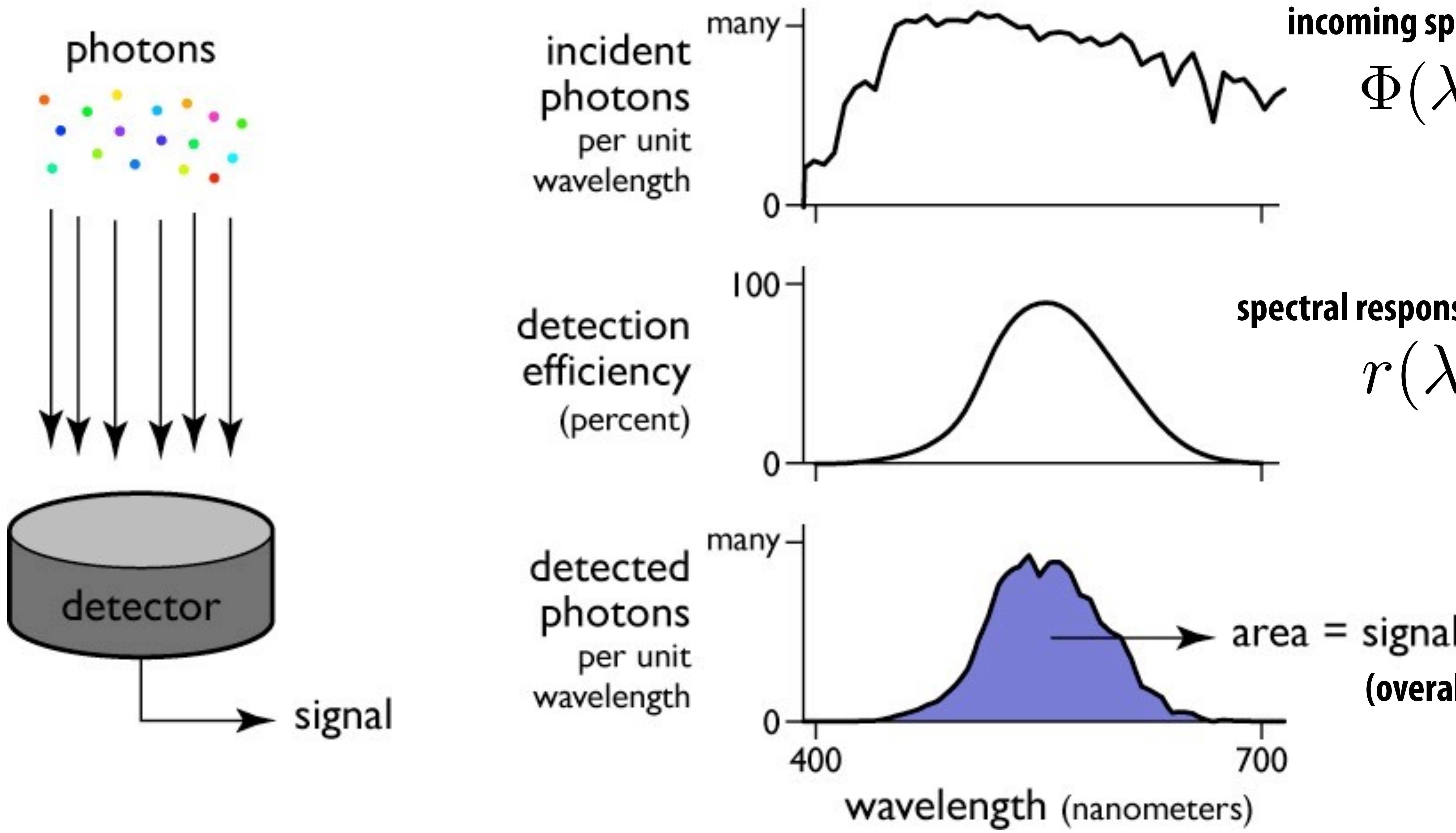
A simple model of a light detector

- Produces a single value (a number) when photons land on it
  - Value depends only on number of photons detected
  - Each photon has a probability of being detected that depends on the wavelength
  - No way to distinguish between signals caused by light of different wavelengths: there is just a number



- This model holds for many detectors:
  - based on semiconductors (e.g., digital cameras)
  - based on visual photopigments (e.g., human eyes)

Simple model of a light detector



incoming spectrum
$\Phi(\lambda)$

spectral response function
$r(\lambda)$

(overall response)

$$R = \int_{\lambda} \Phi(\lambda) r(\lambda) d\lambda$$

Figure credit: Steve Marschner

Dimensionality reduction from ∞ to 1

- **At the detector:**
  - SPD is a function of wavelength (∞ - dimensional signal)
  - Detector result is a scalar value (1 - dimensional signal)

Biological Basis of Color

The eye

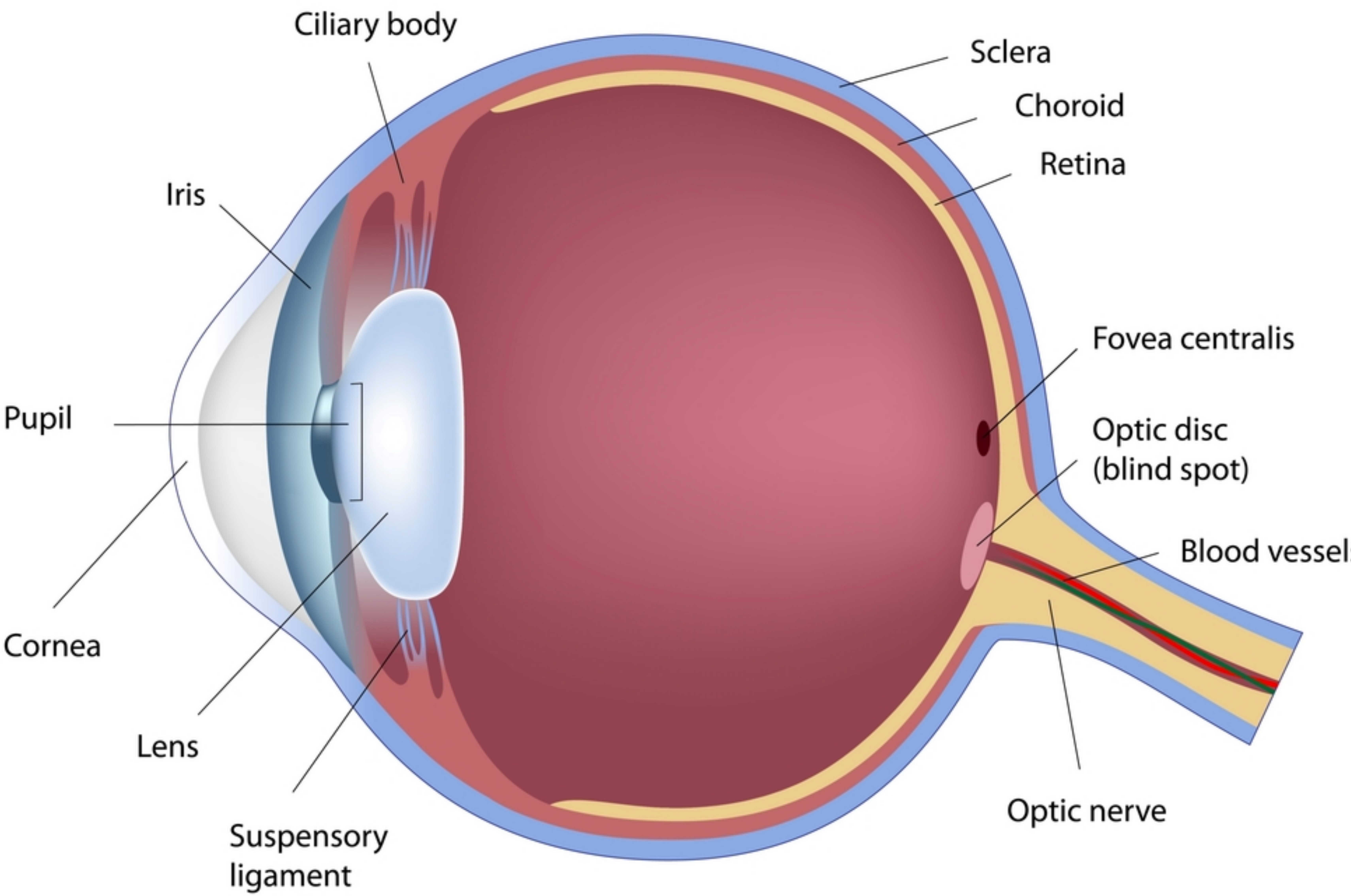


Image credit: Georgia Retina (http://www.garetina.com/about-the-eye)

The eye's photoreceptor cells: rods and cones

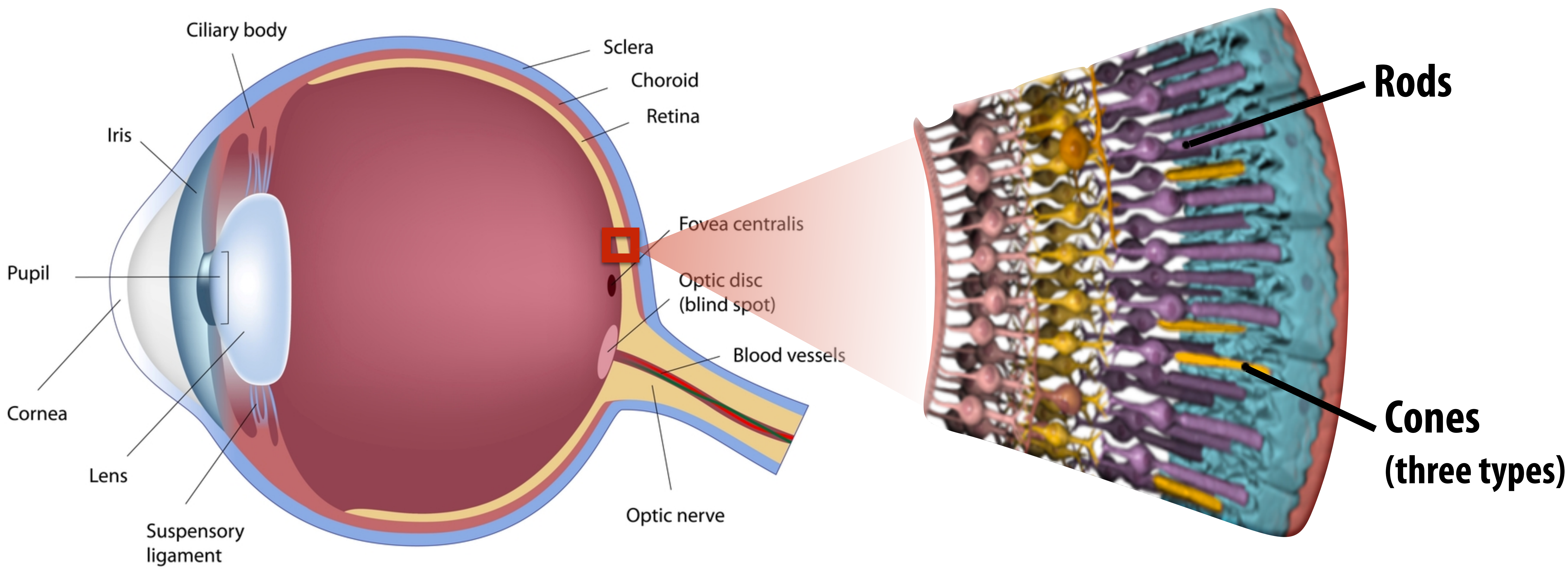


Image credit: http://colorvisionawareness.com/images/pages/eyeDiagram_rodsCones.png

The eye's photoreceptor cells: rods and cones

- Rods are primary receptors under dark viewing conditions (scotopic conditions)
  - Approx. 120 million rods in human eye
  - Sense light intensity (shades of gray, not color)
- Cones are primary receptors under high-light viewing conditions (photopic conditions, e.g., daylight)
  - Approx. 6-7 million cones in the human eye
  - There are three types of cones
  - Each of the three types of cone feature a different "spectral response". This will be critical to color vision (more on this soon…)

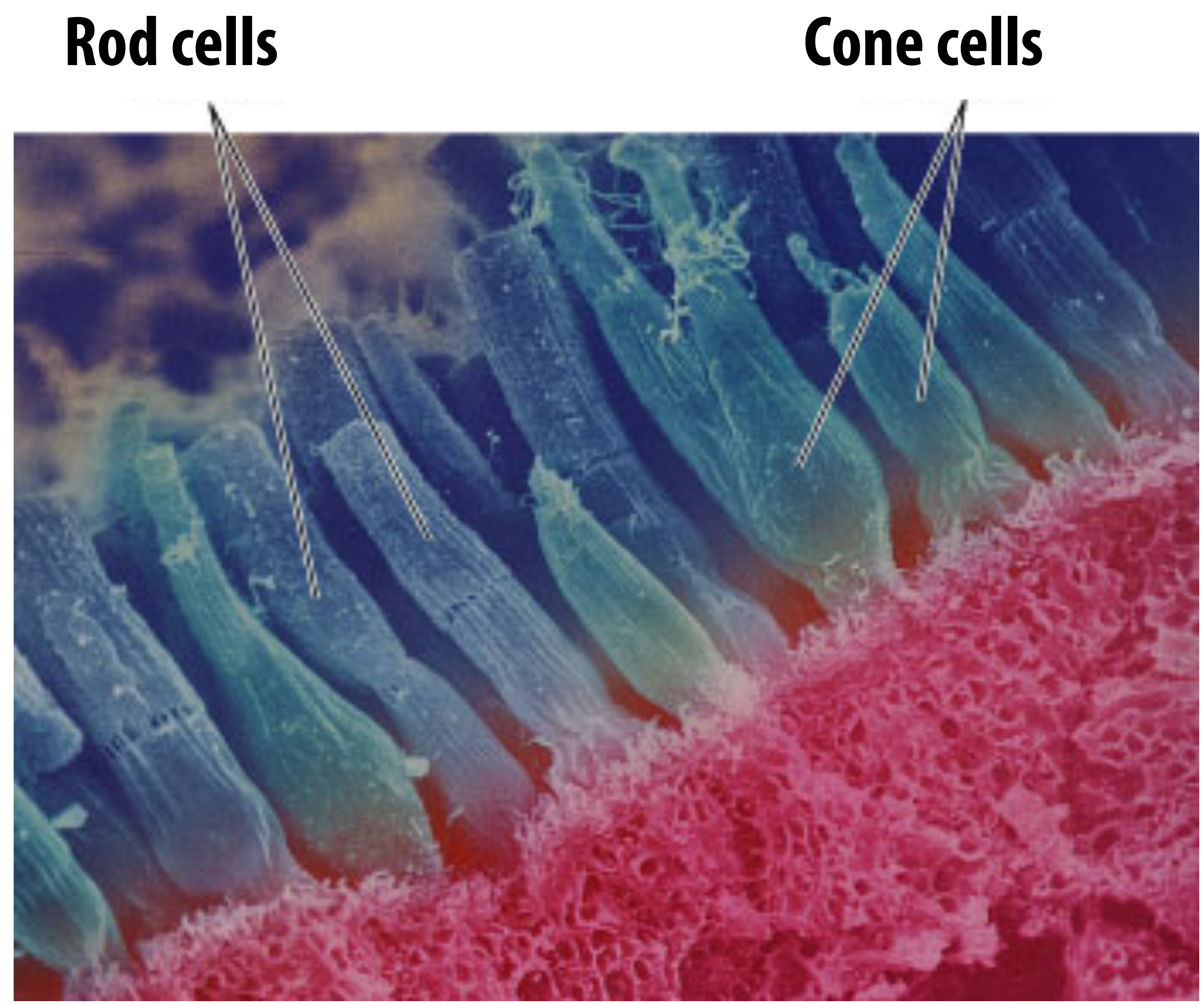
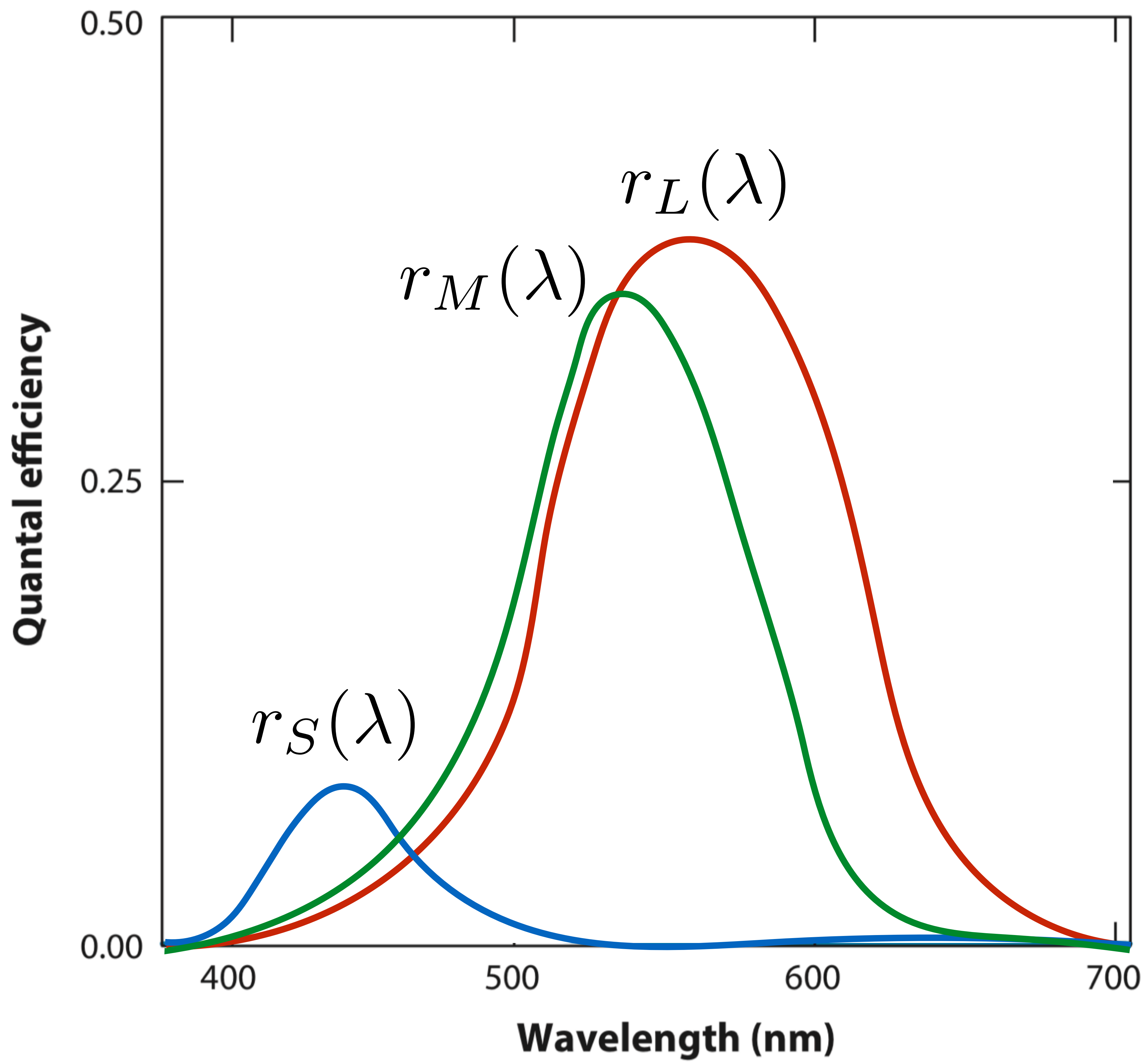


Figure credit: http://ebooks.bfwpub.com/life.php Figure 45.18

Human retinal cone cell response functions
(L, M, S types)

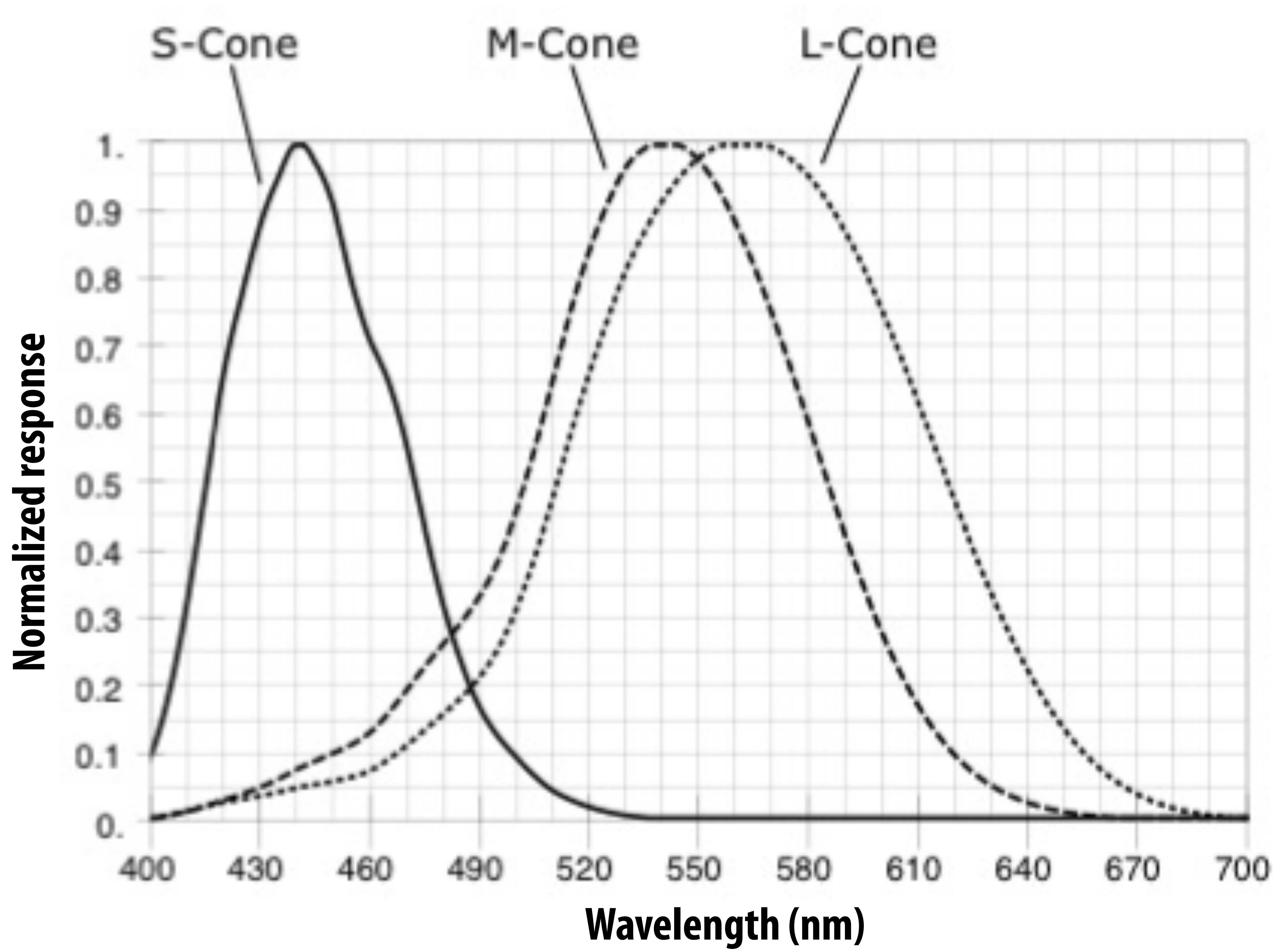
Three types of cone cells: S, M, and L (corresponding to peak response at short, medium, and long wavelengths)

Probability that a photon will cause a photopigment isomerization

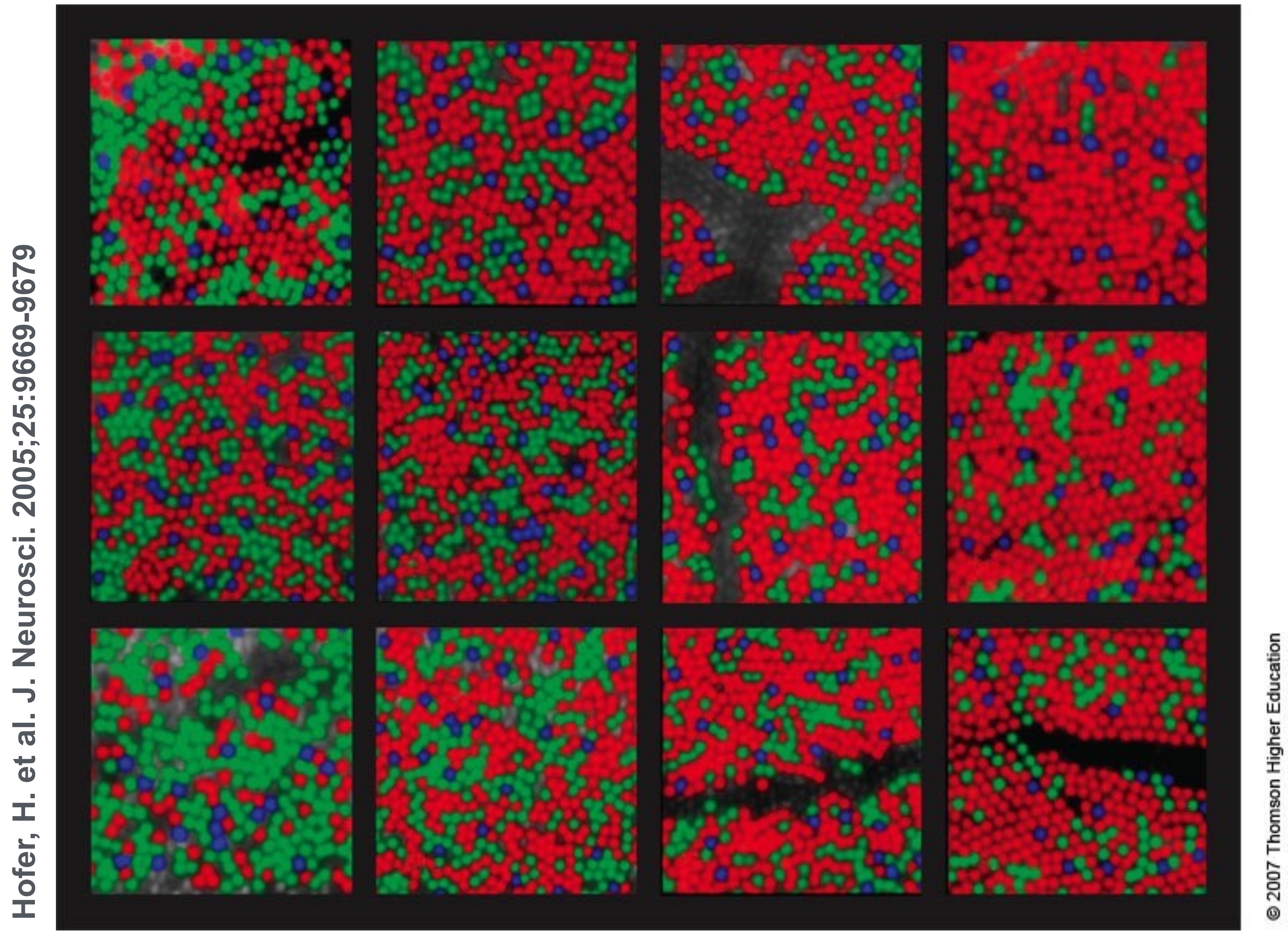


Brainard, Color and the Cone Mosaic, 2015.

Human retinal cone cell normalized response functions (L, M, S types)



Fraction of three cone cell types varies widely



Distribution of cone cells at edge of fovea in 12 different humans with normal color vision. Note high variability of percentage of different cone cell types. (false color image)

Spectral response of cones

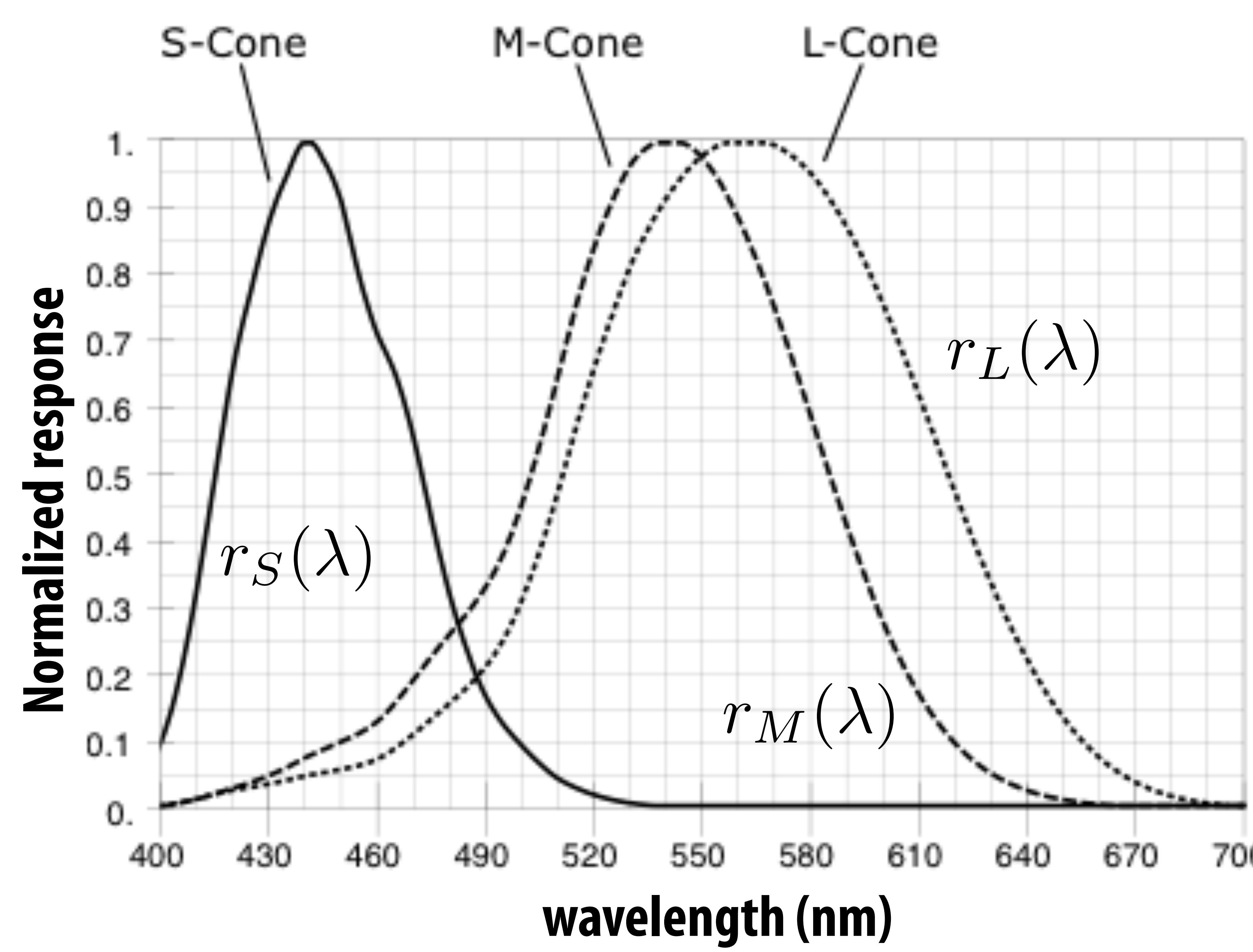
Three types of cones: S, M, and L cones (corresponding to peak response at short, medium, and long wavelengths)

$$S = \int_\lambda \Phi(\lambda) r_S(\lambda) d\lambda$$

$$M = \int_\lambda \Phi(\lambda) r_M(\lambda) d\lambda$$

$$L = \int_\lambda \Phi(\lambda) r_L(\lambda) d\lambda$$

Response functions for S, M, and L cones



In other words:

Your eye measures three values: S, M, L

Spectral response of cones (discrete form)

Three types of cones: S, M, and L cones (corresponding to peak response at short, medium, and long wavelengths)

Discrete form: written as vector dot products:
(now using *s* to denote discrete representation of SPD $\Phi(\lambda)$)

$$S = r_S \cdot s$$

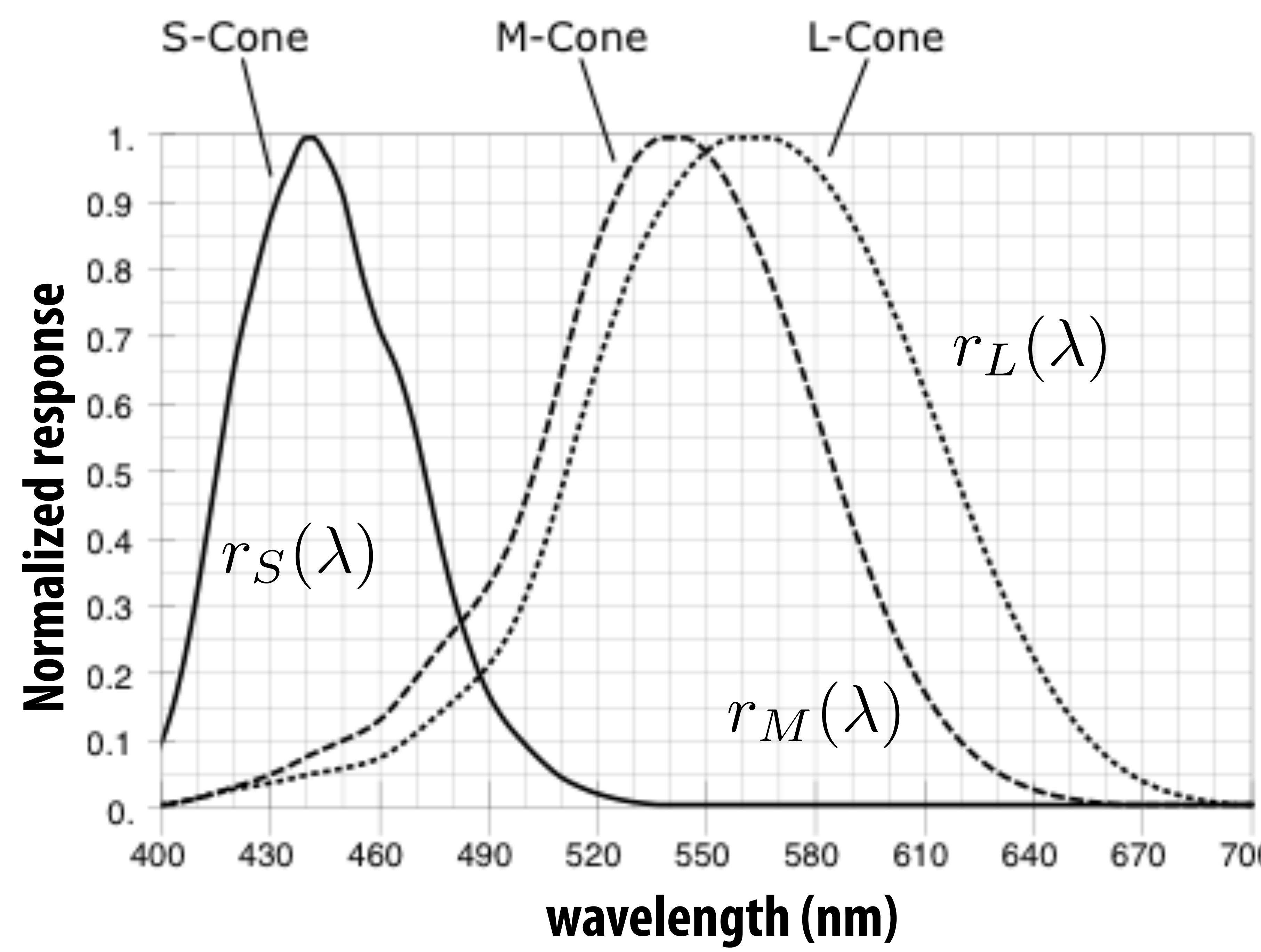
$$M = r_M \cdot s$$

$$L = r_L \cdot s$$

Matrix formulation:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Response functions for S, M, and L cones



Example: spectral response of human cone cells

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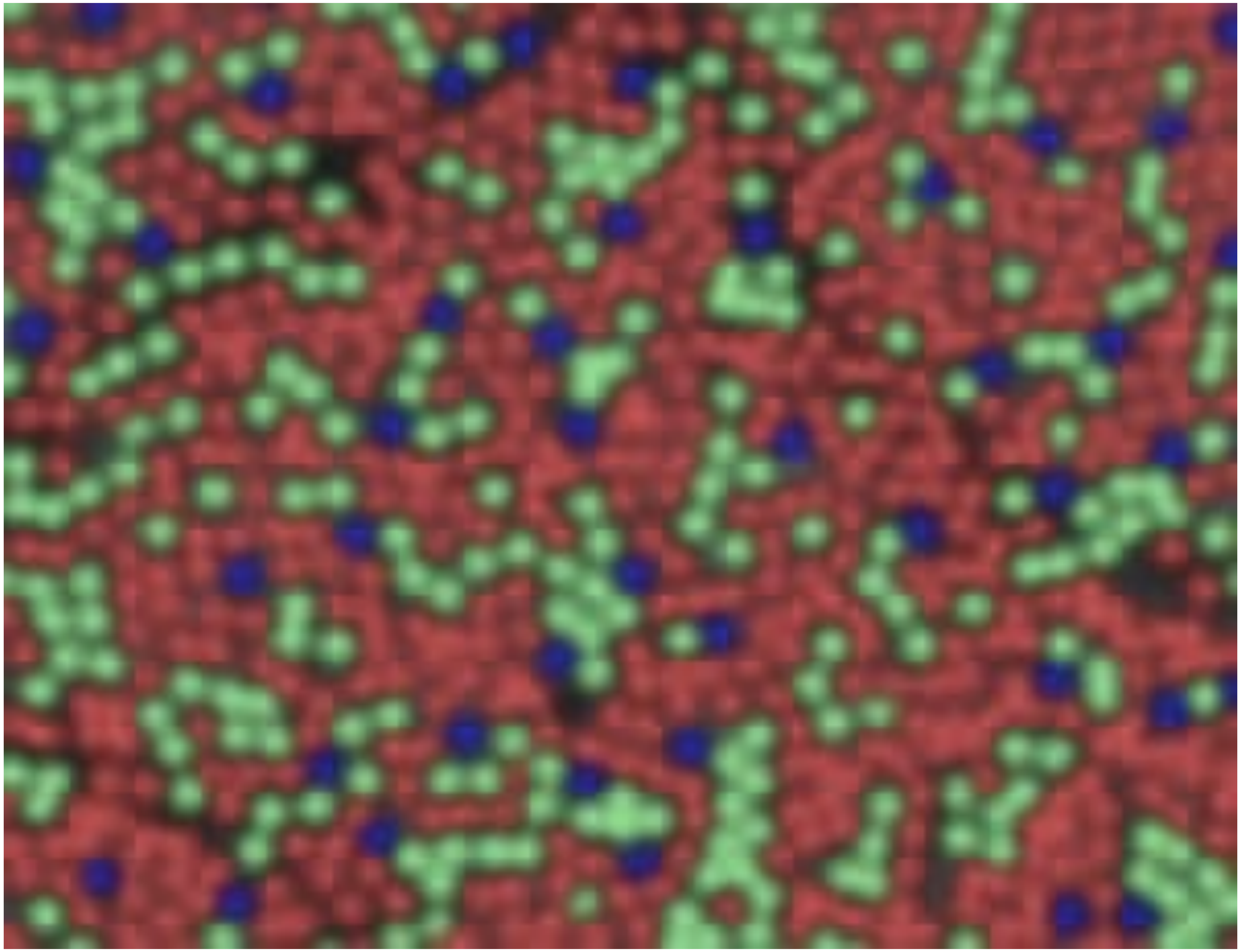
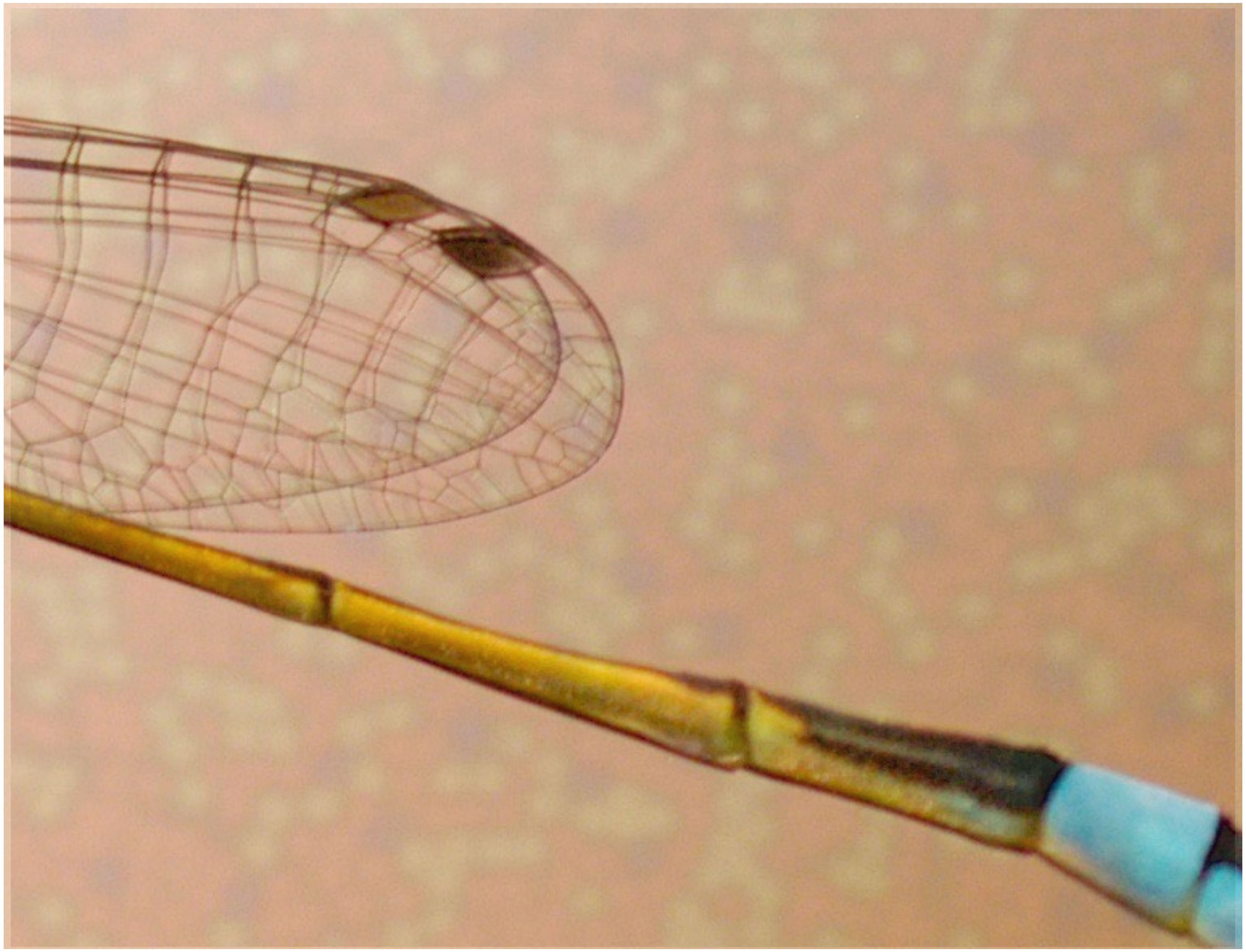


Scene projected onto retina

Credit: Sabesan, http://depts.washington.edu/sabaolab/

Example: spectral response of human cone cells



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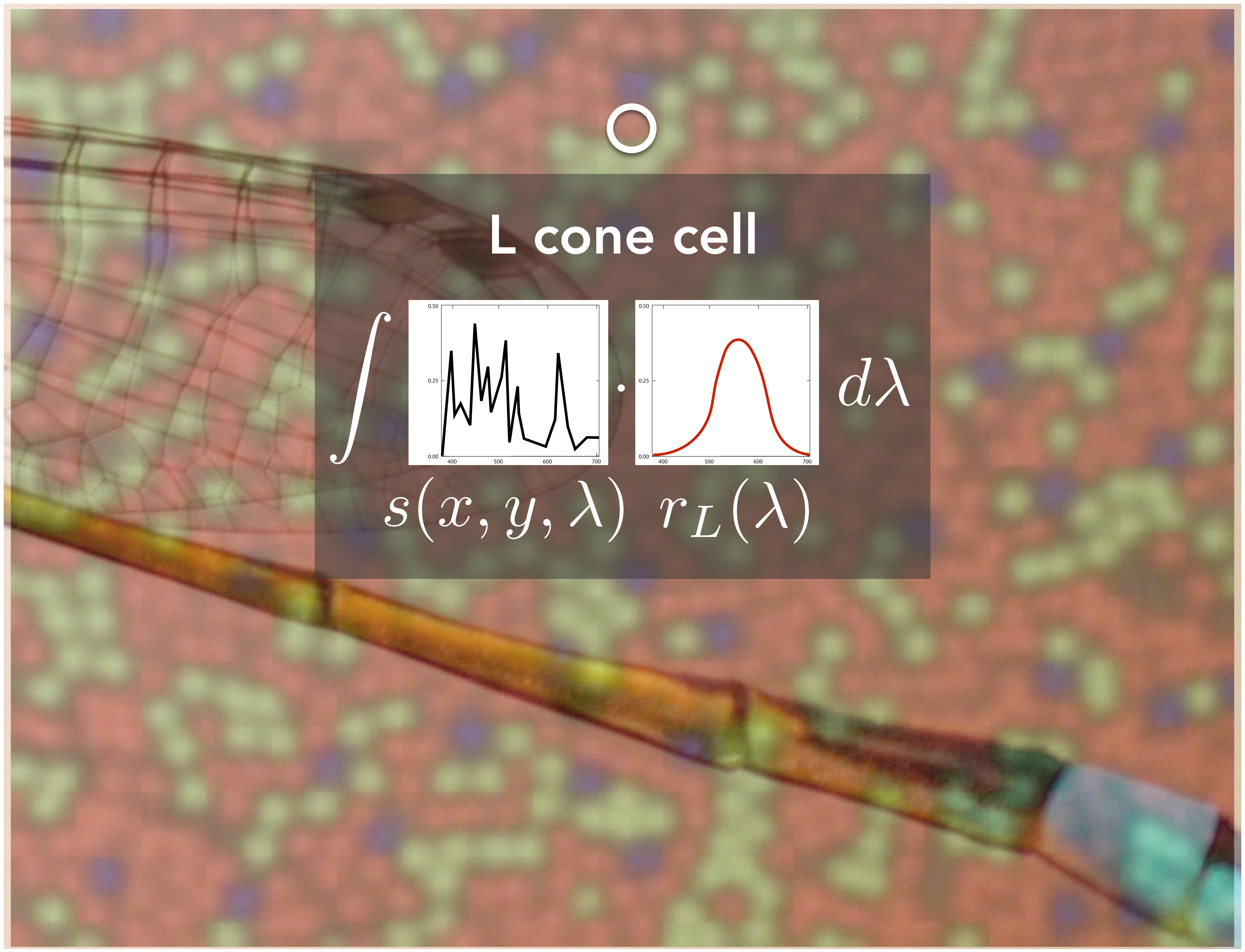
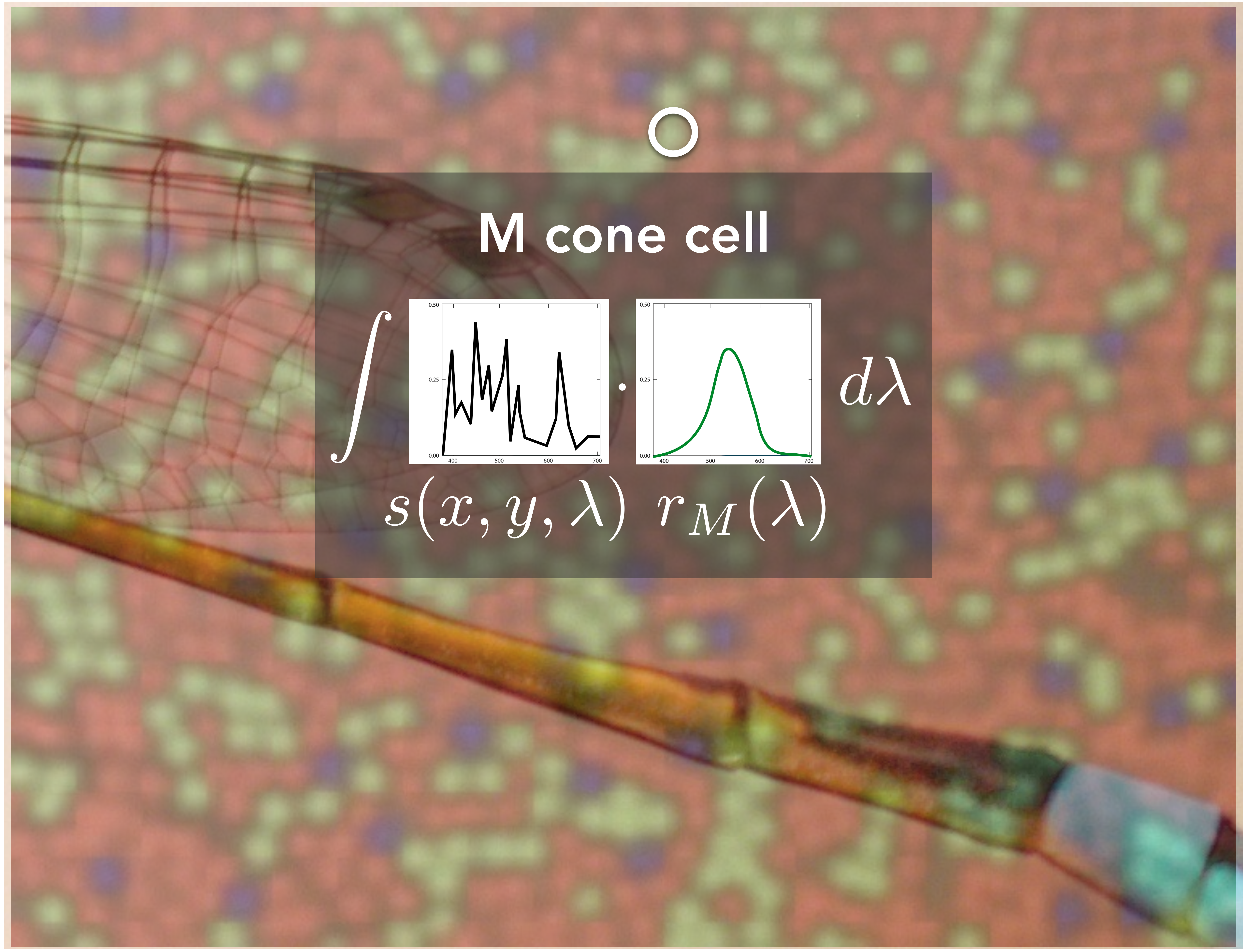
Example: spectral response of human cone cells



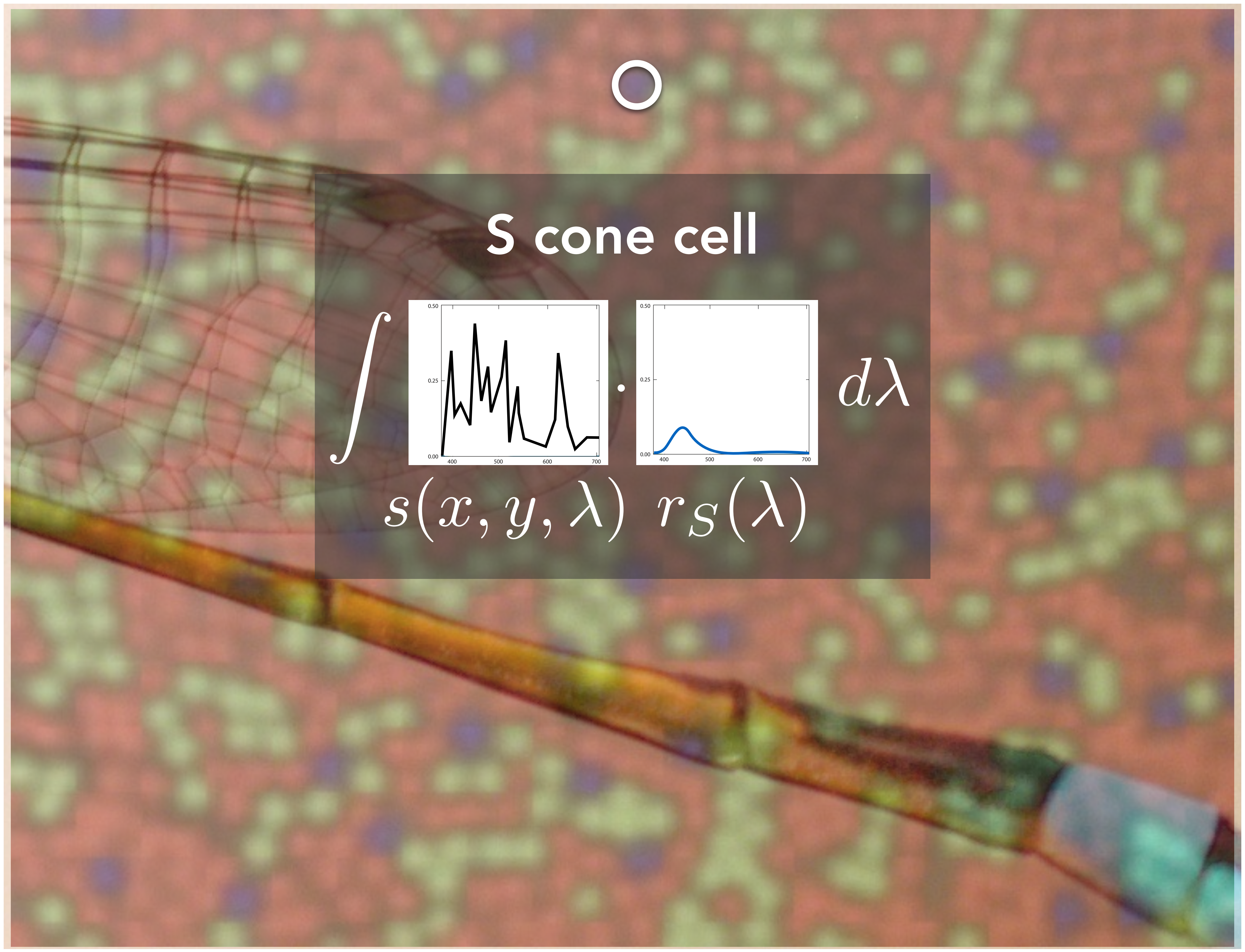
Credit: Sabesan, http://depts.washington.edu/sabaolab

Example: spectral response of human cone cells



Credit: Sabesan, http://depts.washington.edu/sabaolab

Example: spectral response of human cone cells



Credit: Sabesan, http://depts.washington.edu/sabaolab

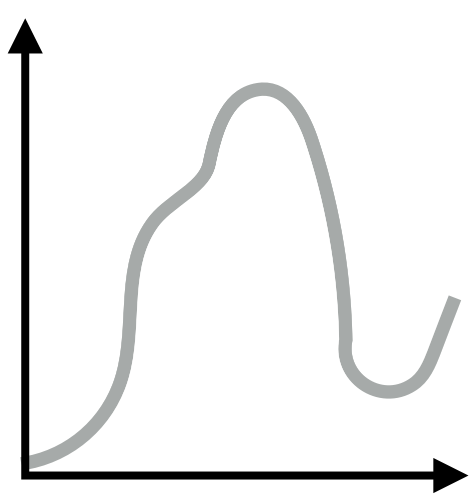
Dimensionality reduction from ∞ to 3

- **At each position on the human retina:**
  - SPD is a function of wavelength (∞ - dimensional signal)
  - 3 types of cones near that position produce three scalar values (3 - dimensional signal)

- **What about 2D images?**
  - The dimensionality reduction described above is happening at every 2D position in our visual field

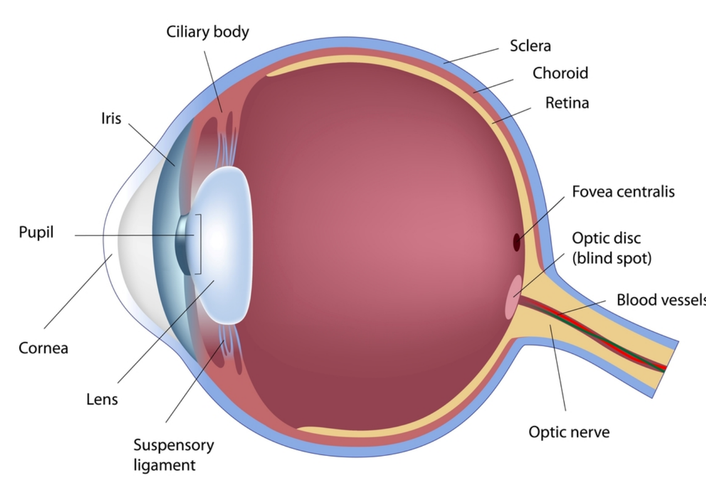
The human visual system

- **Human eye does not directly measure the spectrum of incoming light**
  - **a.k.a. the brain does not receive "a spectrum" from the eye**
- **The eye measures three response values = (S, M, L). The result of integrating the incoming spectrum against response functions of S, M, L-cones**



Spectrum

$\Phi(\lambda)$



Eye

Focuses light on retina
Cones measure light
(photopic case)

cone responses (S, M, L) carried along optic nerve

Brain

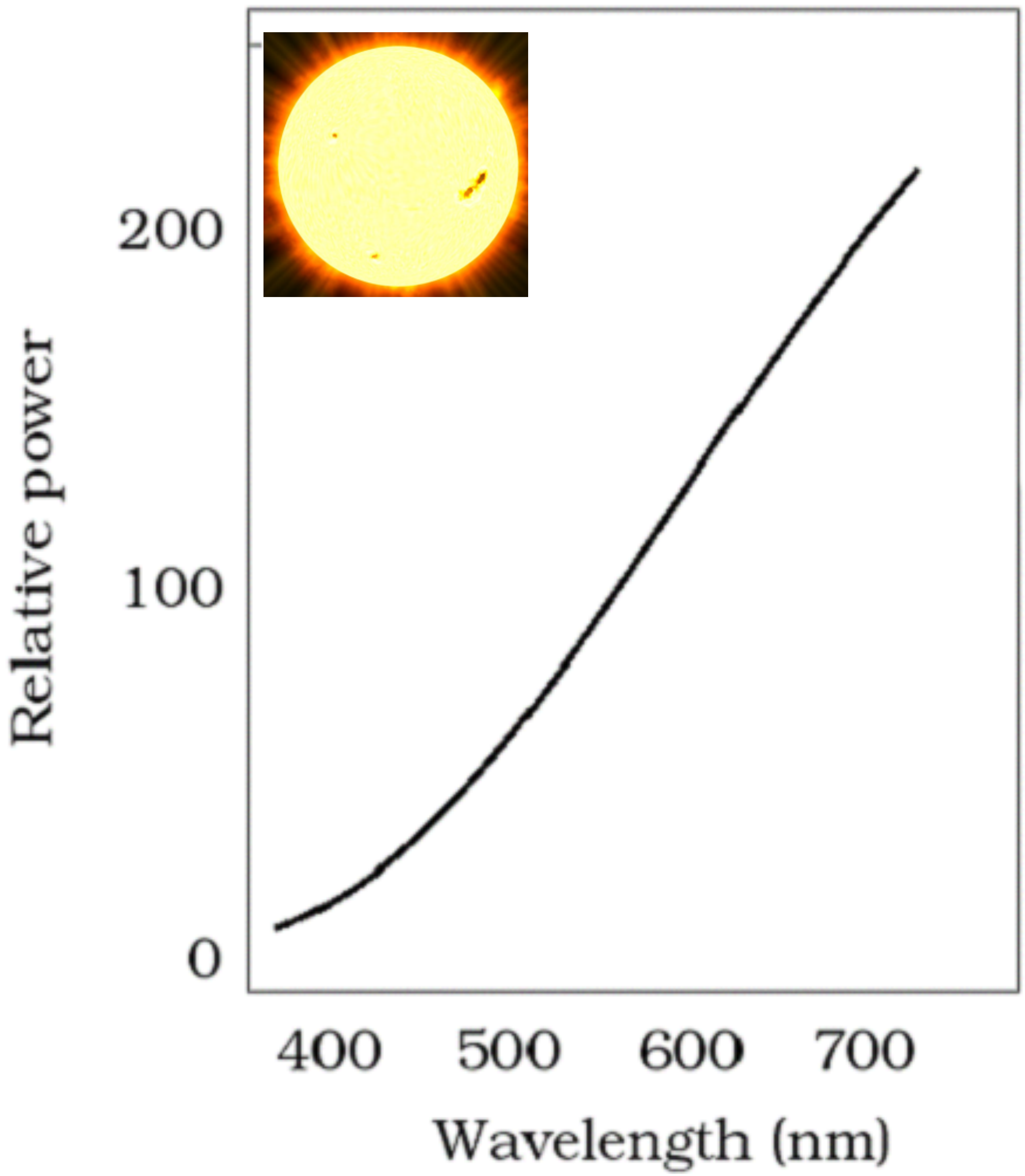
Metamerism

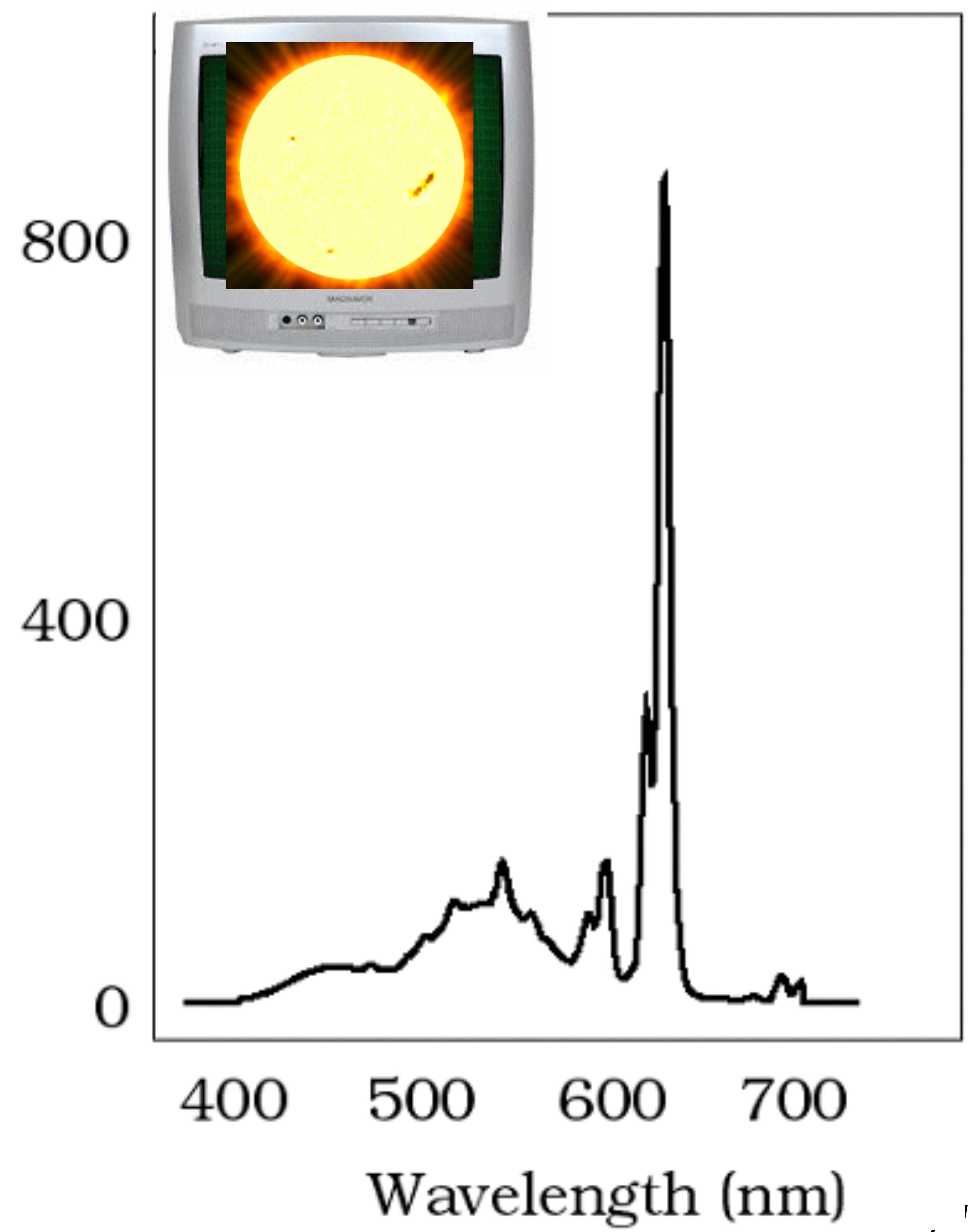
Metamers

- Metameters are two different spectra (∞-dim) that project to the same (S,M,L) (3-dim) response.
  - These will appear to have the same color to a human
- The existence of metamers is critical to color reproduction
  - Don't have to reproduce the full spectrum of a real world scene
  - Example: A metamer can reproduce the perceived color of a real-world scene on a display with pixels of only three colors

Metamerism

Color matching is an important illusion that is understood quantitatively





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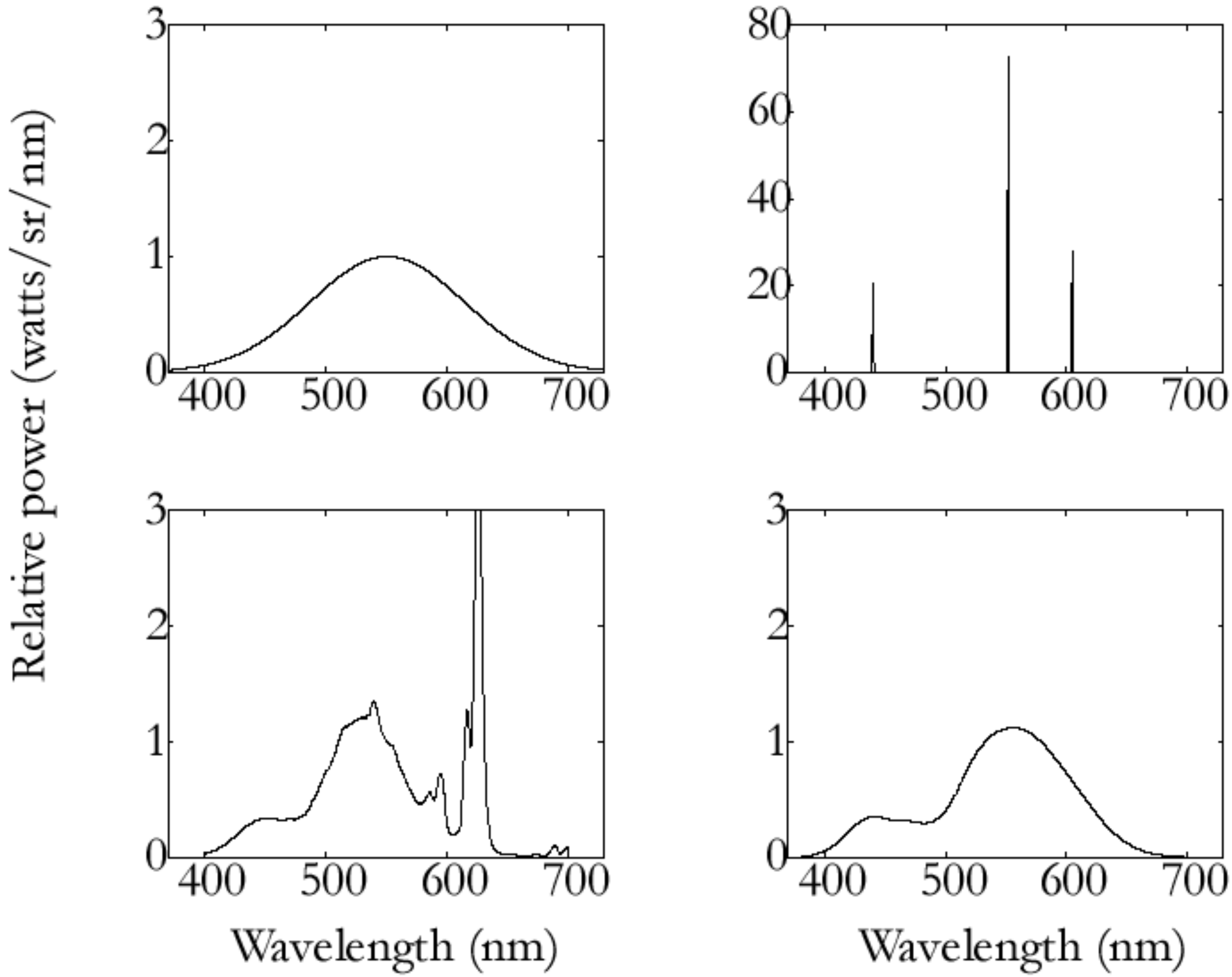
Metamerism is a big effect



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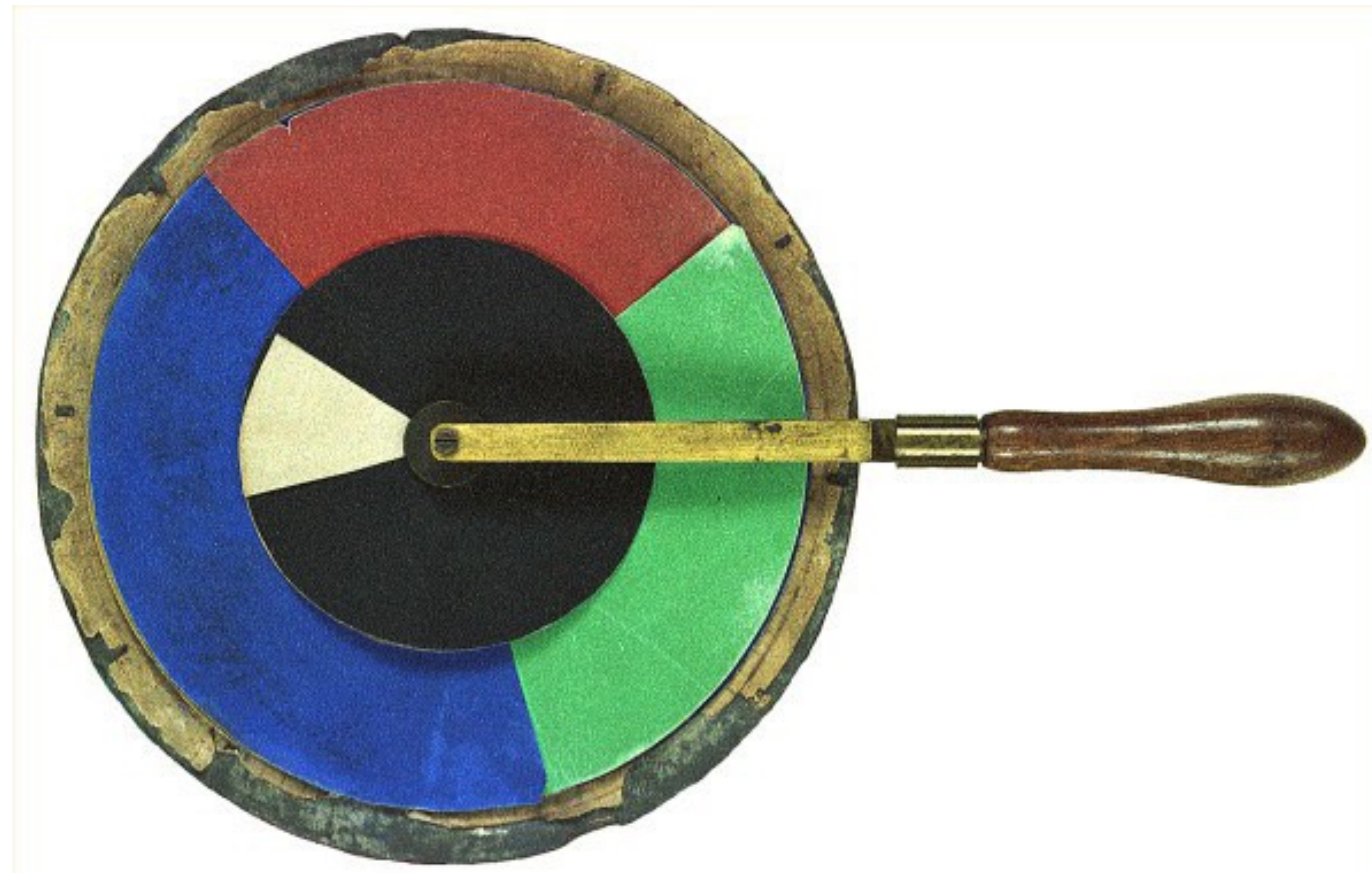
Searching for a Basis for Colors: Color Matching Experiment

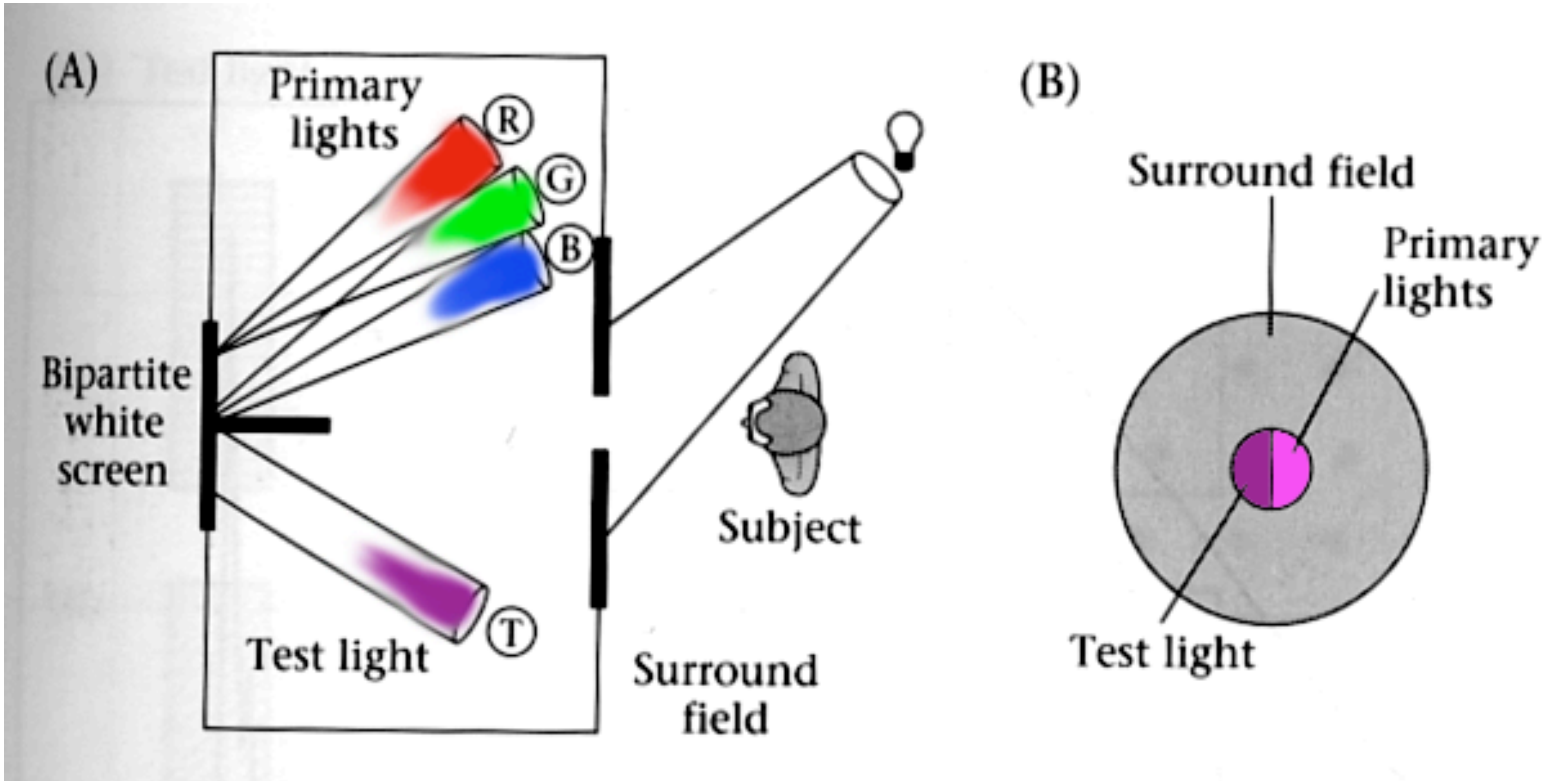
Maxwell's crucial color matching experiment



http://designblog.rietveldacademie.nl/?p=68422

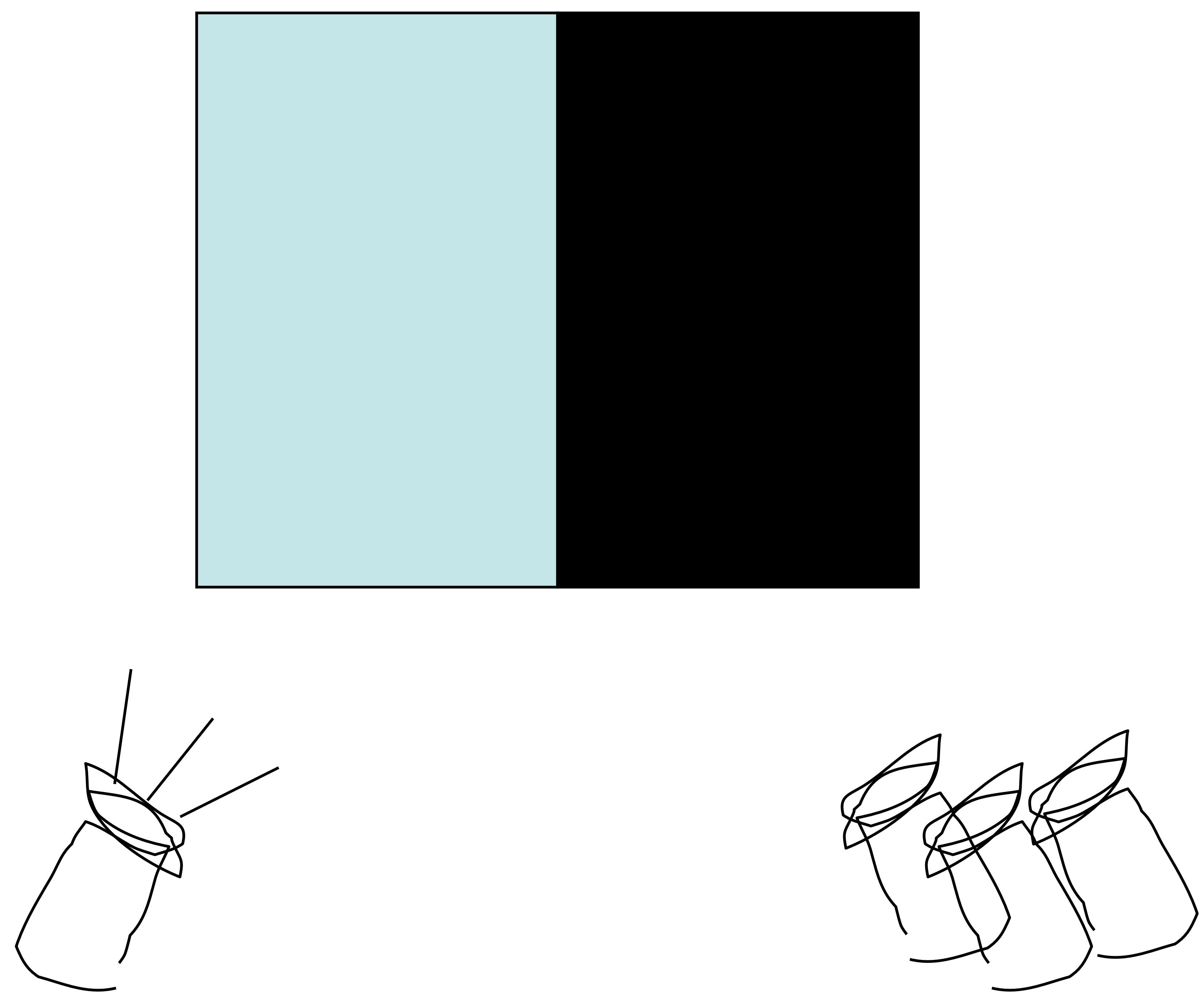
Portrait: http://rsta.royalsocietypublishing.org/content/366/1871/1685

Color matching experiment



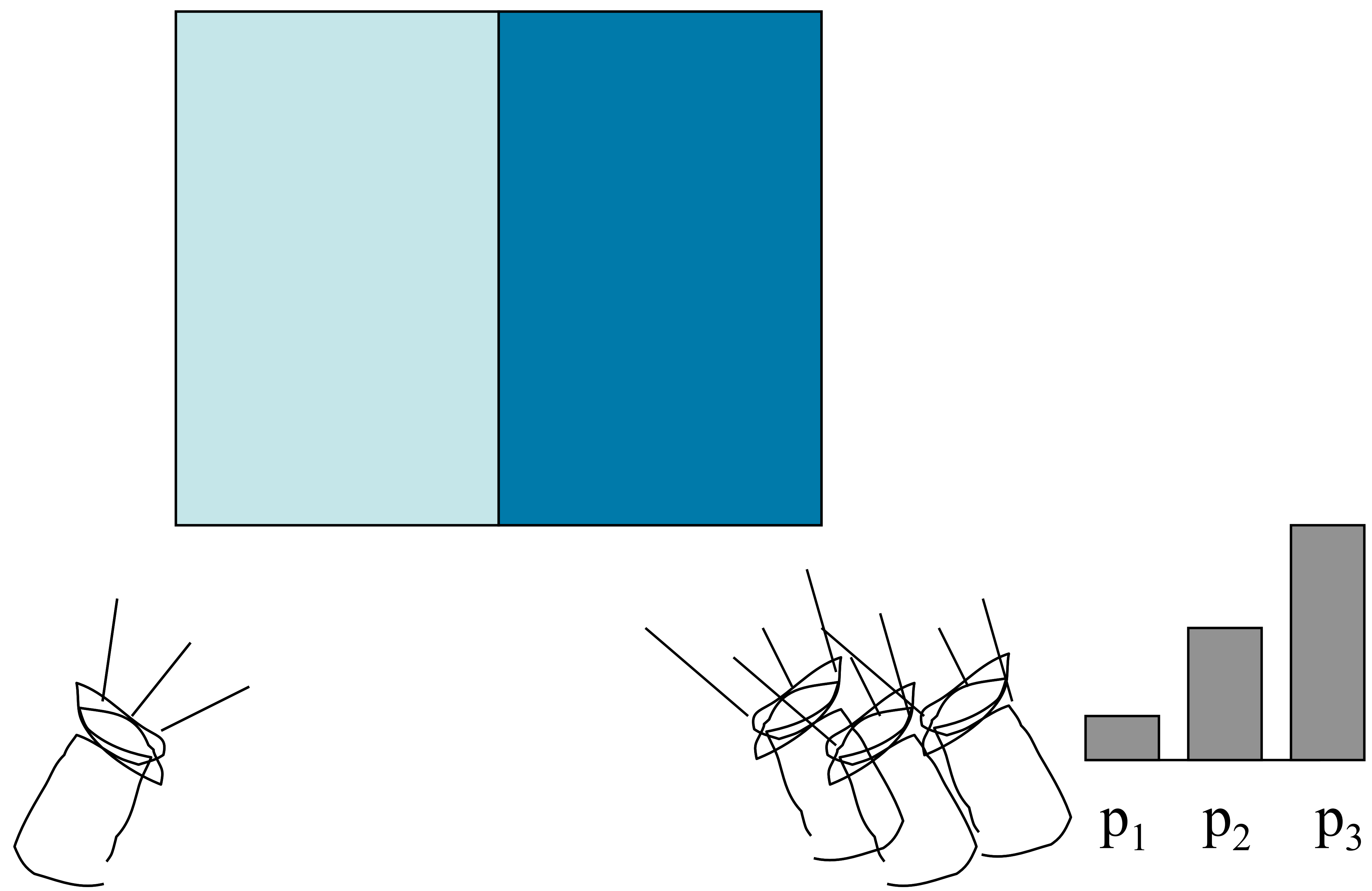
- Same idea as spinning top, fancier implementation (Maxwell did this too)
- Show test light spectrum on left
- Mix "primaries" on right until they match
- The primaries need not be RGB

Example experiment



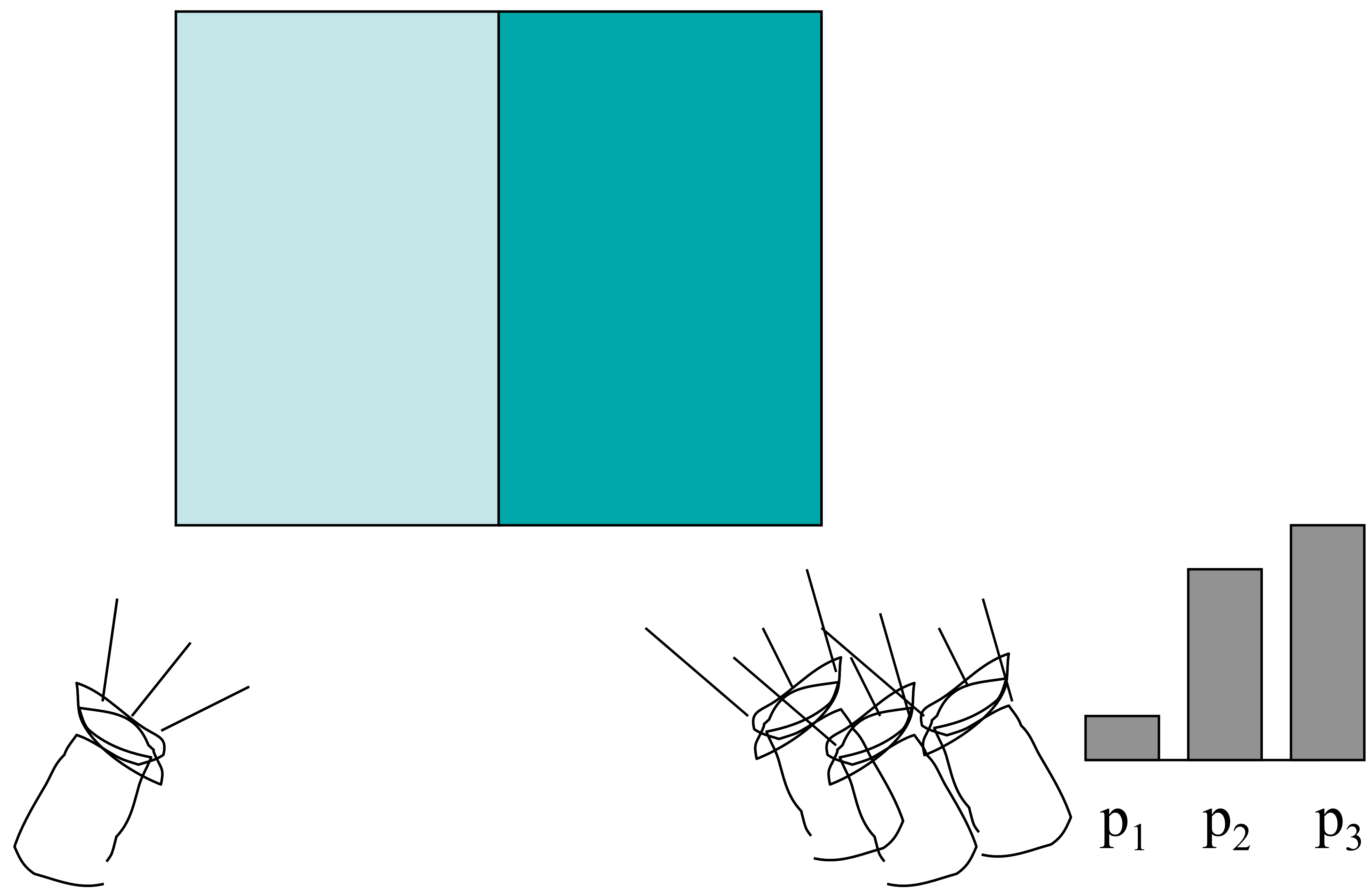
Slide from Durand
and Freeman 06

Example experiment



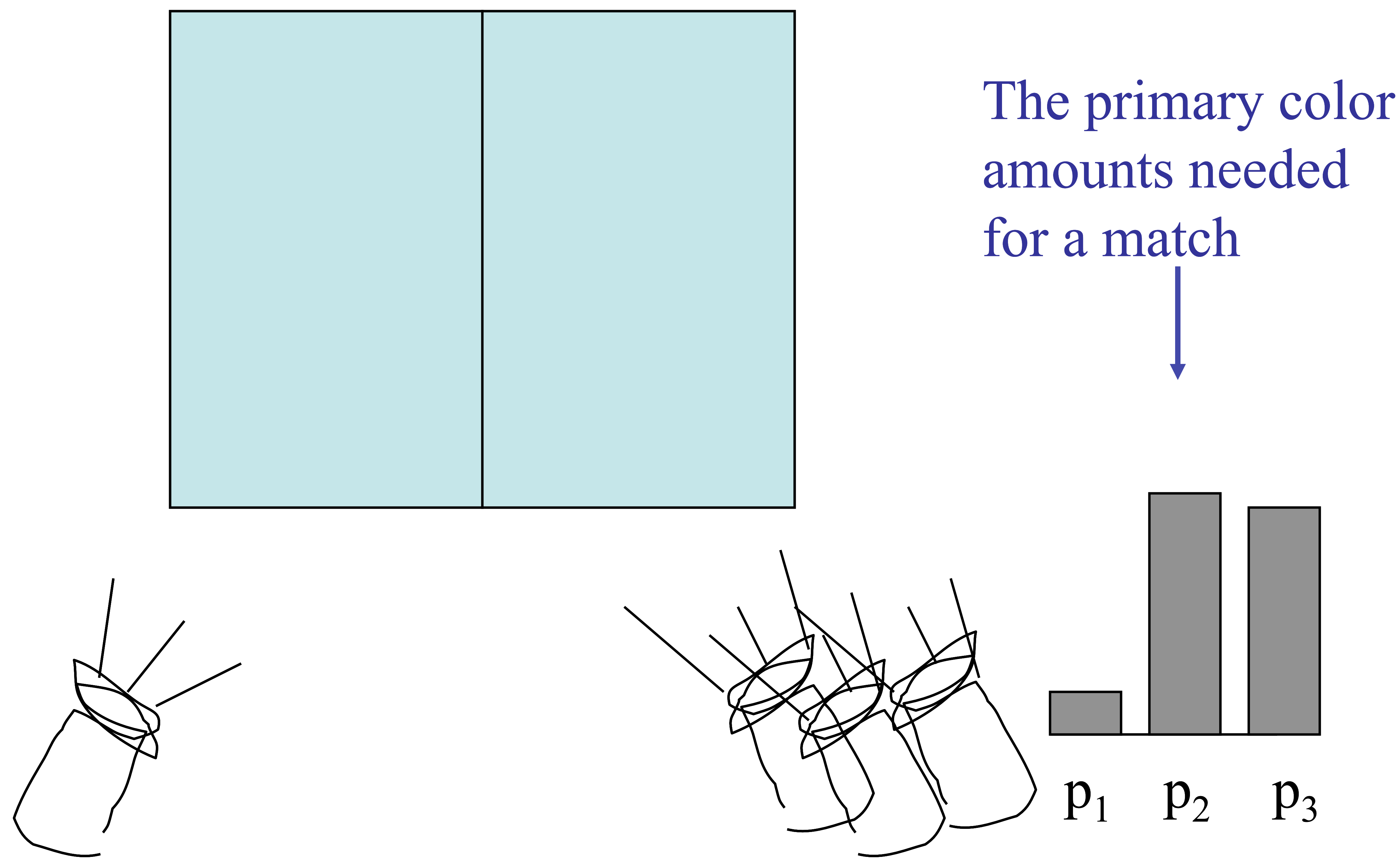
Slide from Durand and Freeman 06

Example experiment



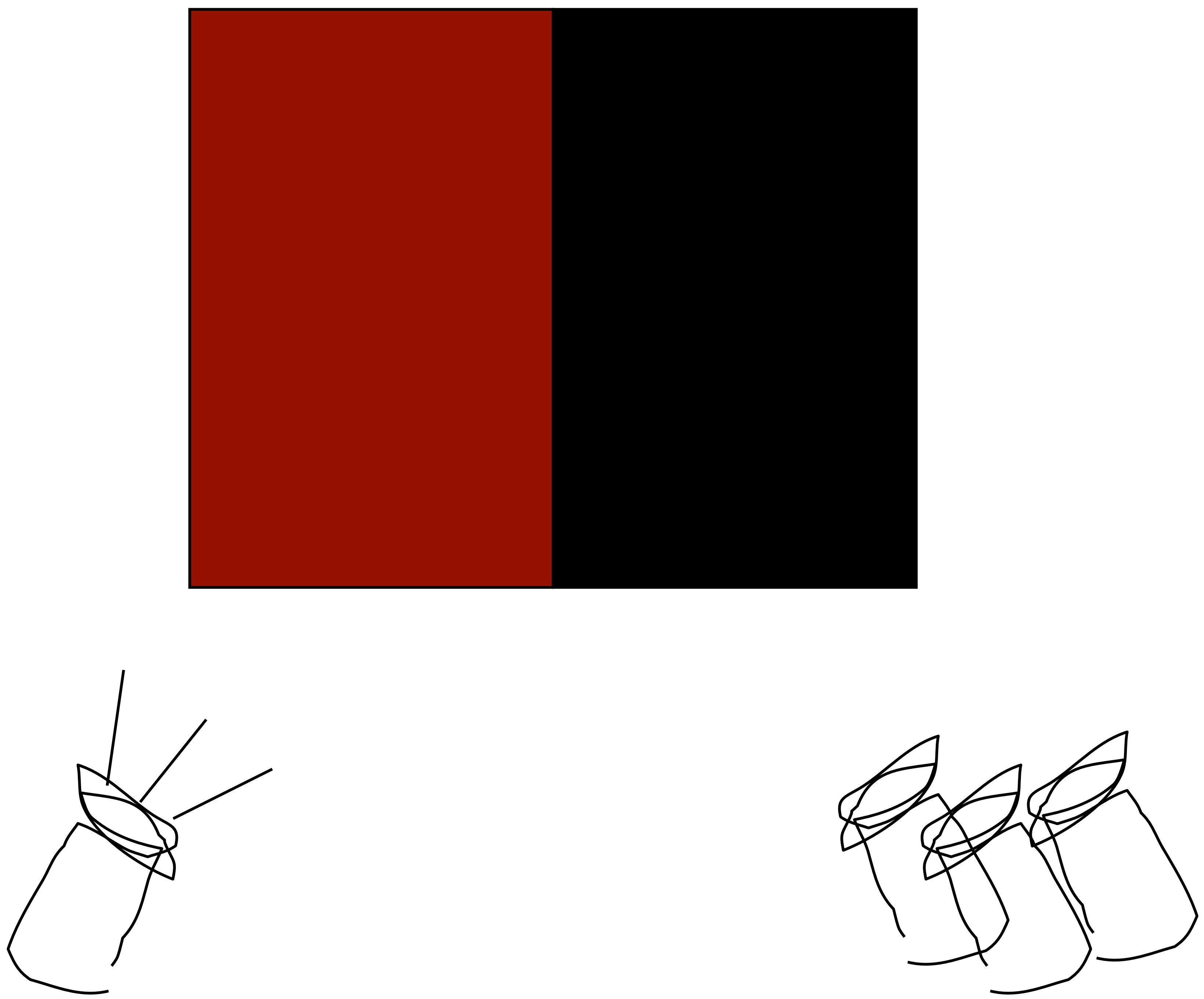
Slide from Durand and Freeman 06

Example experiment



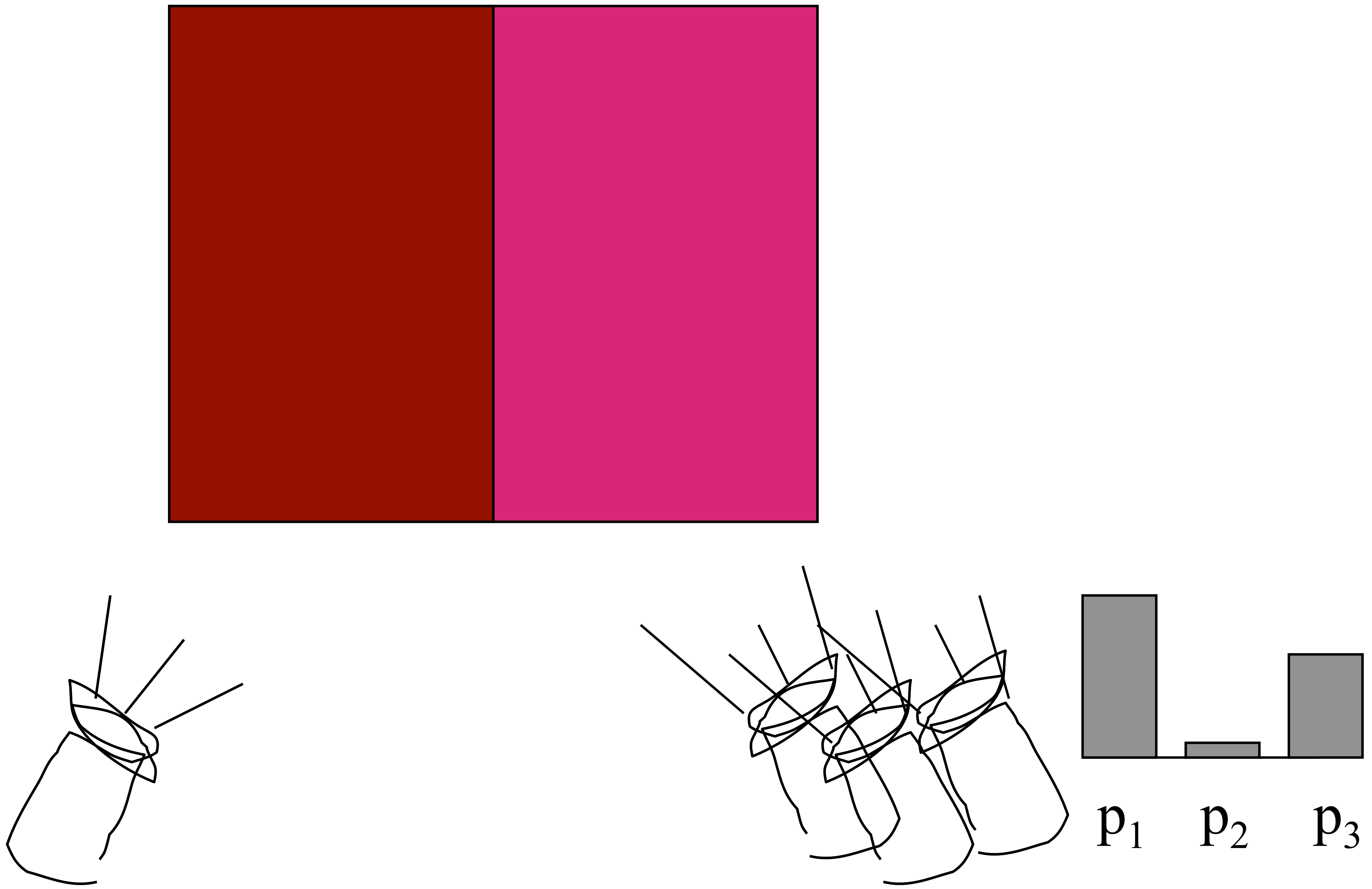
Slide from Durand and Freeman 06

Experiment 2: out of gamut target



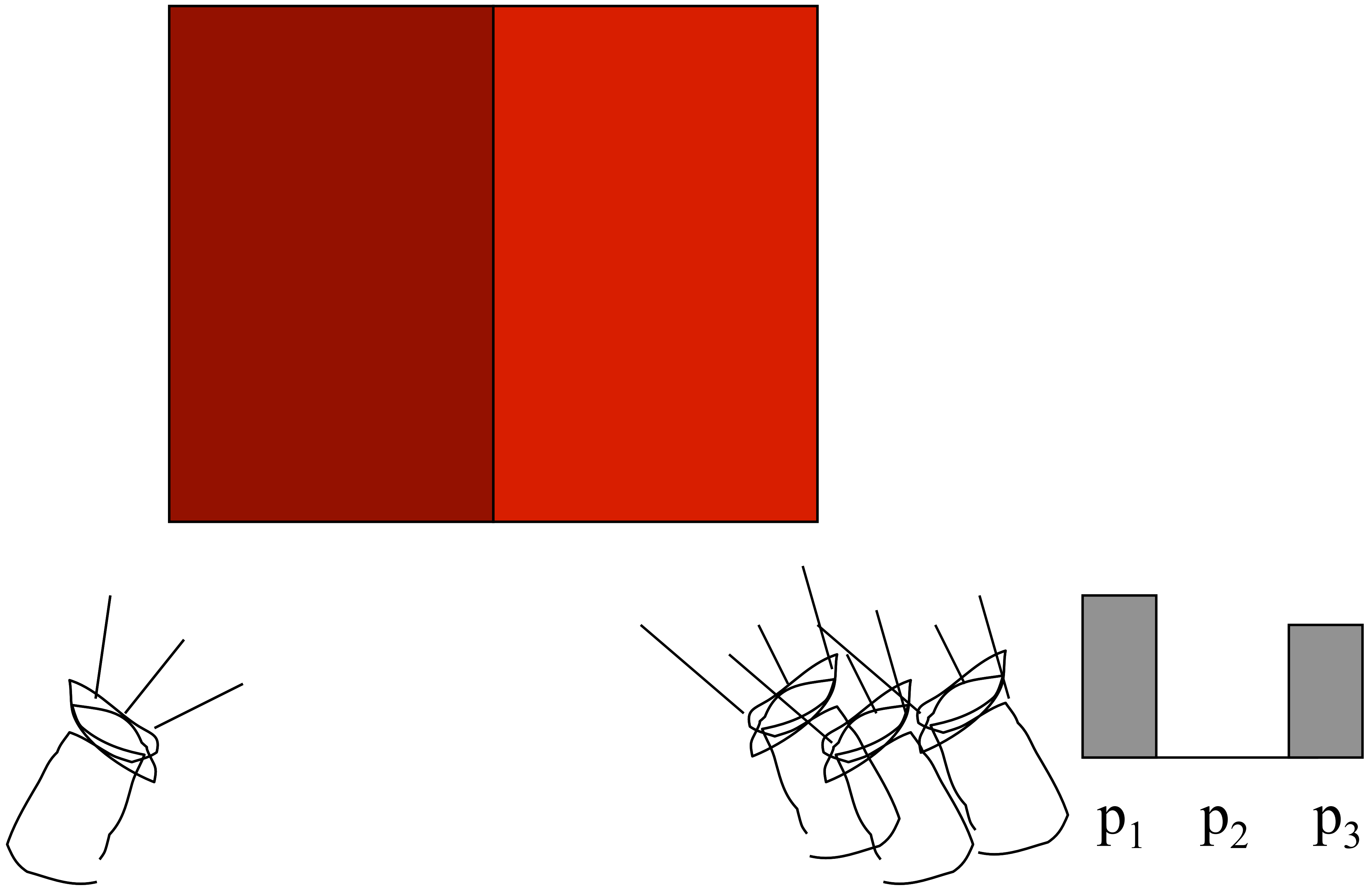
Slide from Durand and Freeman 06

Experiment 2: out of gamut target



Slide from Durand and Freeman 06

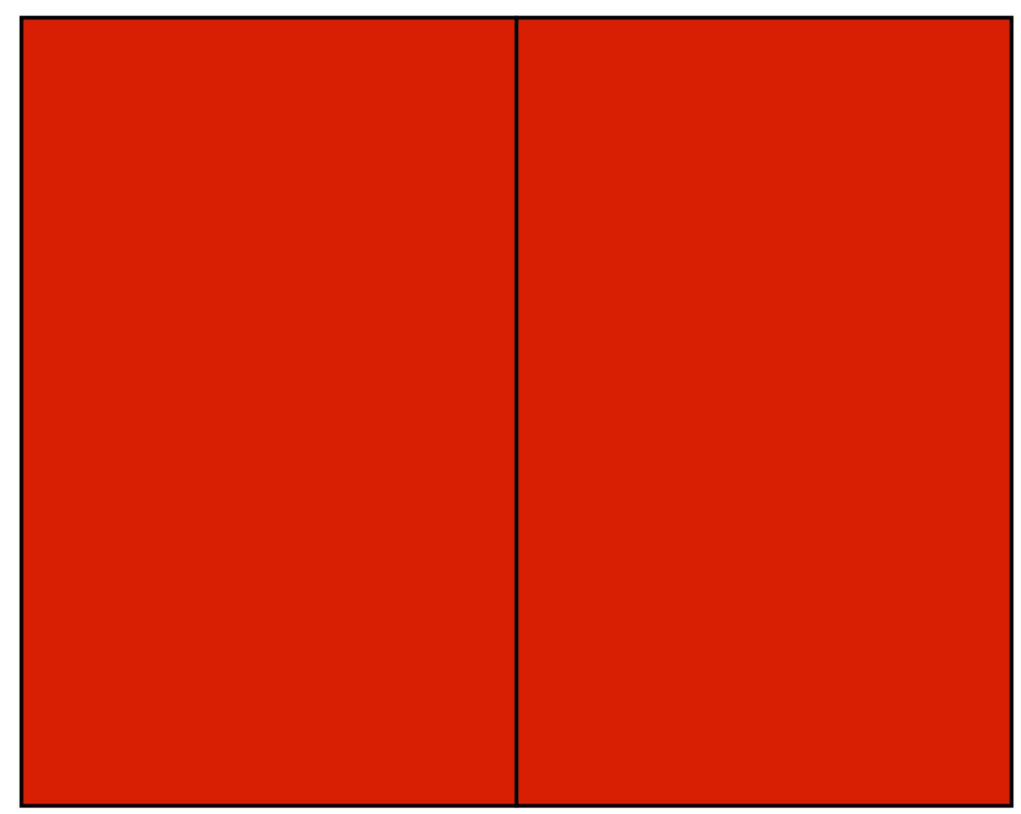
Experiment 2: out of gamut target



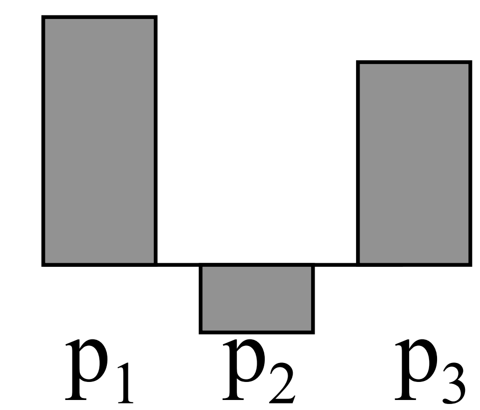
Slide from Durand
and Freeman 06

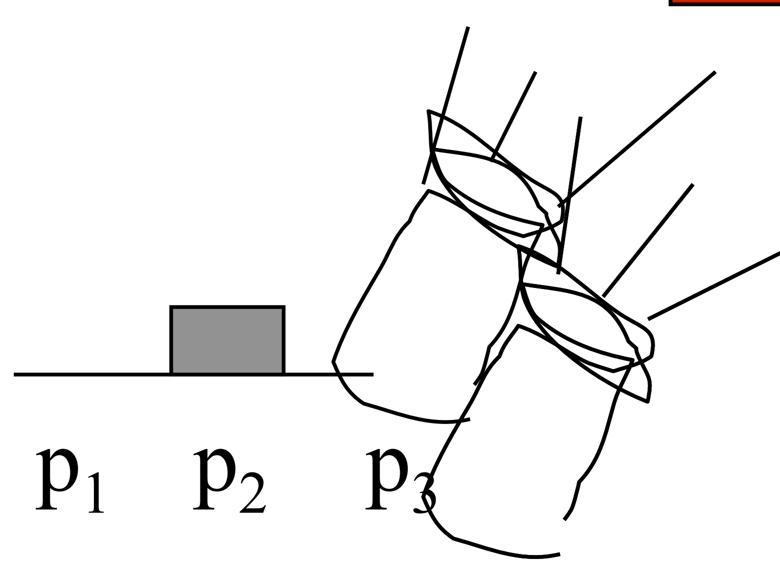
Experiment 2: out of gamut target

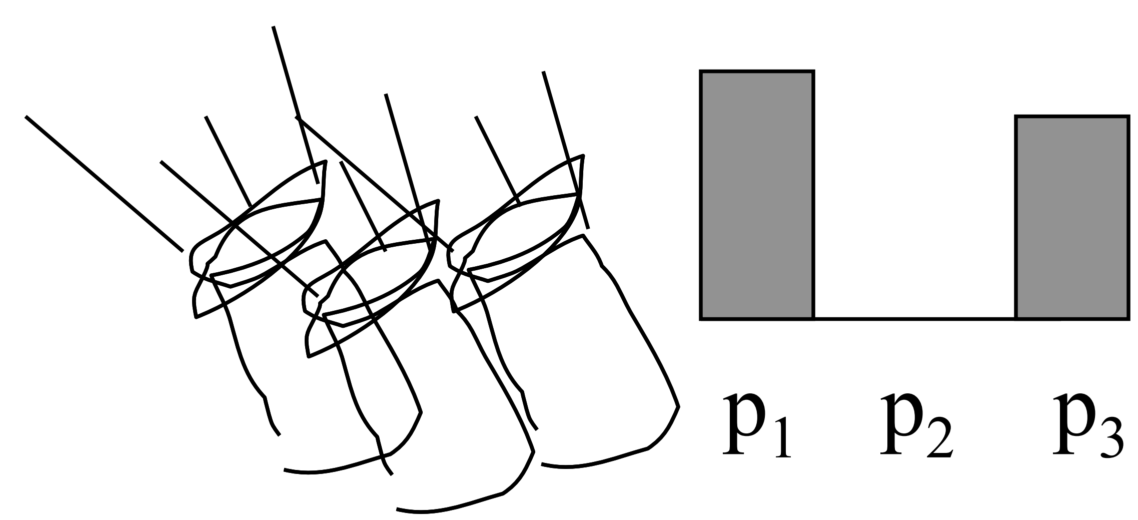
We say a "negative" amount of p_2 was needed to make the match, because we added it to the test color's side.



The primary color amounts needed for a match:



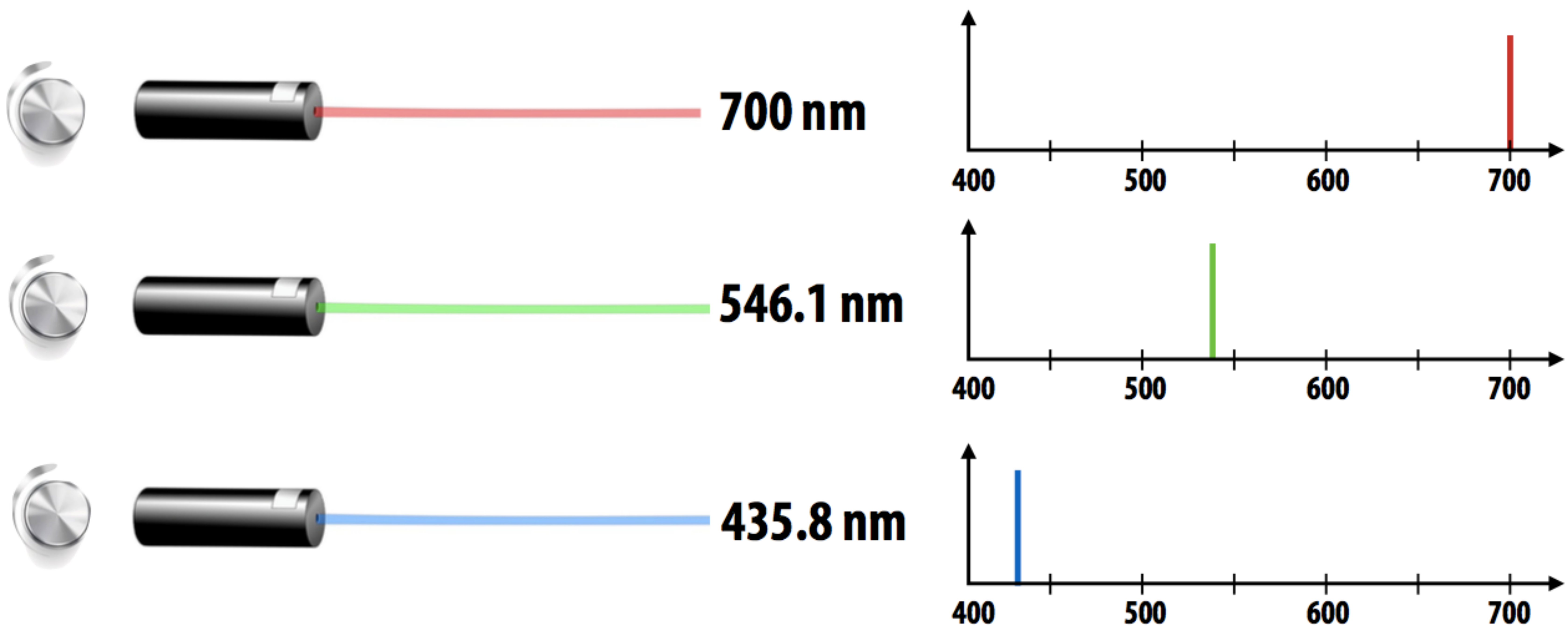




Slide from Durand and Freeman 06

CIE RGB color matching experiment

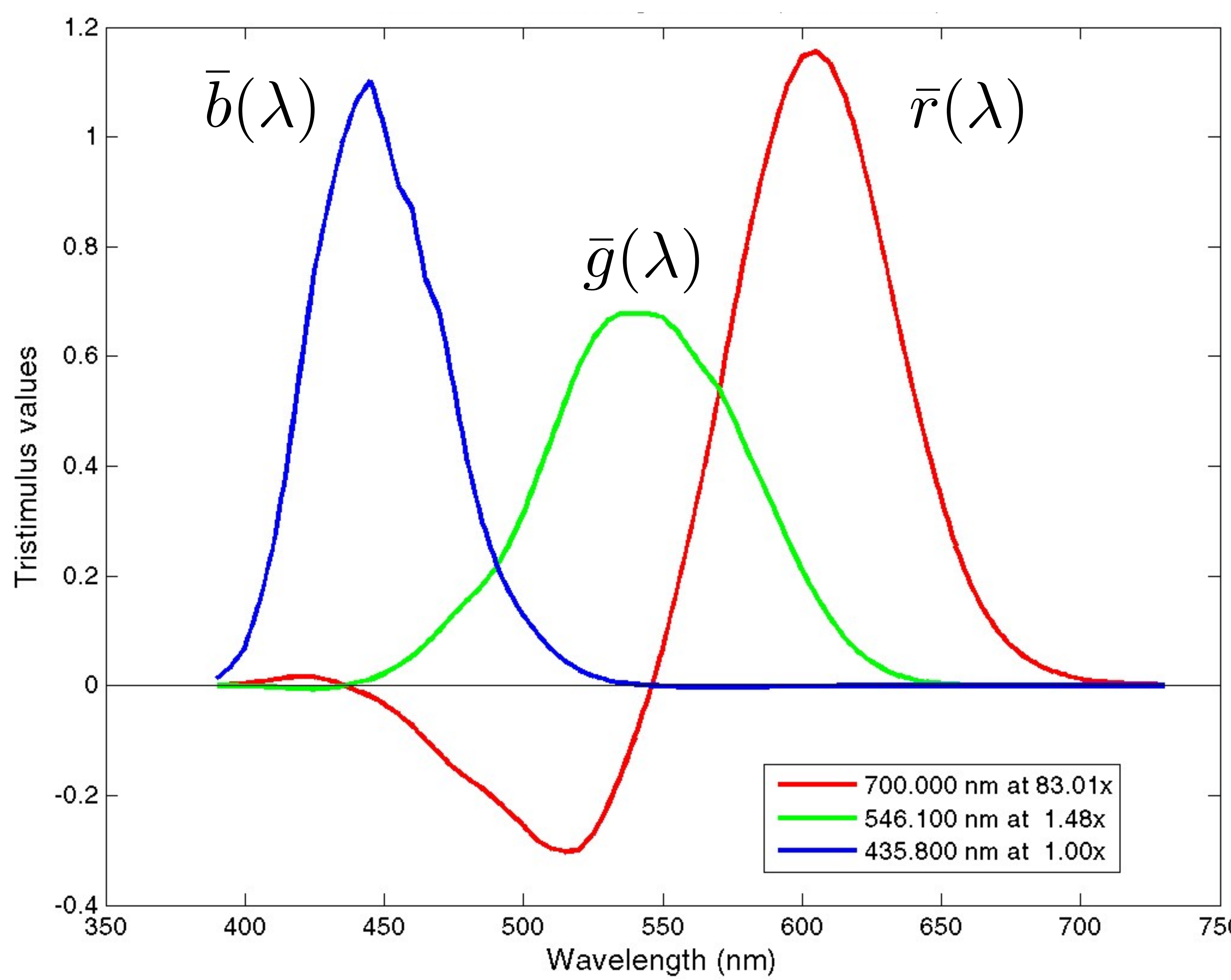
Same setup as additive color matching before, but primaries are monochromatic light (single wavelength) of the following wavelengths defined by CIE RGB standard



The test light is also a monochromatic light

CIE RGB color matching functions

This graph plots how much of each CIE RGB primary light must be combined to match the appearance of a monochromatic light of the wavelength given on x-axis



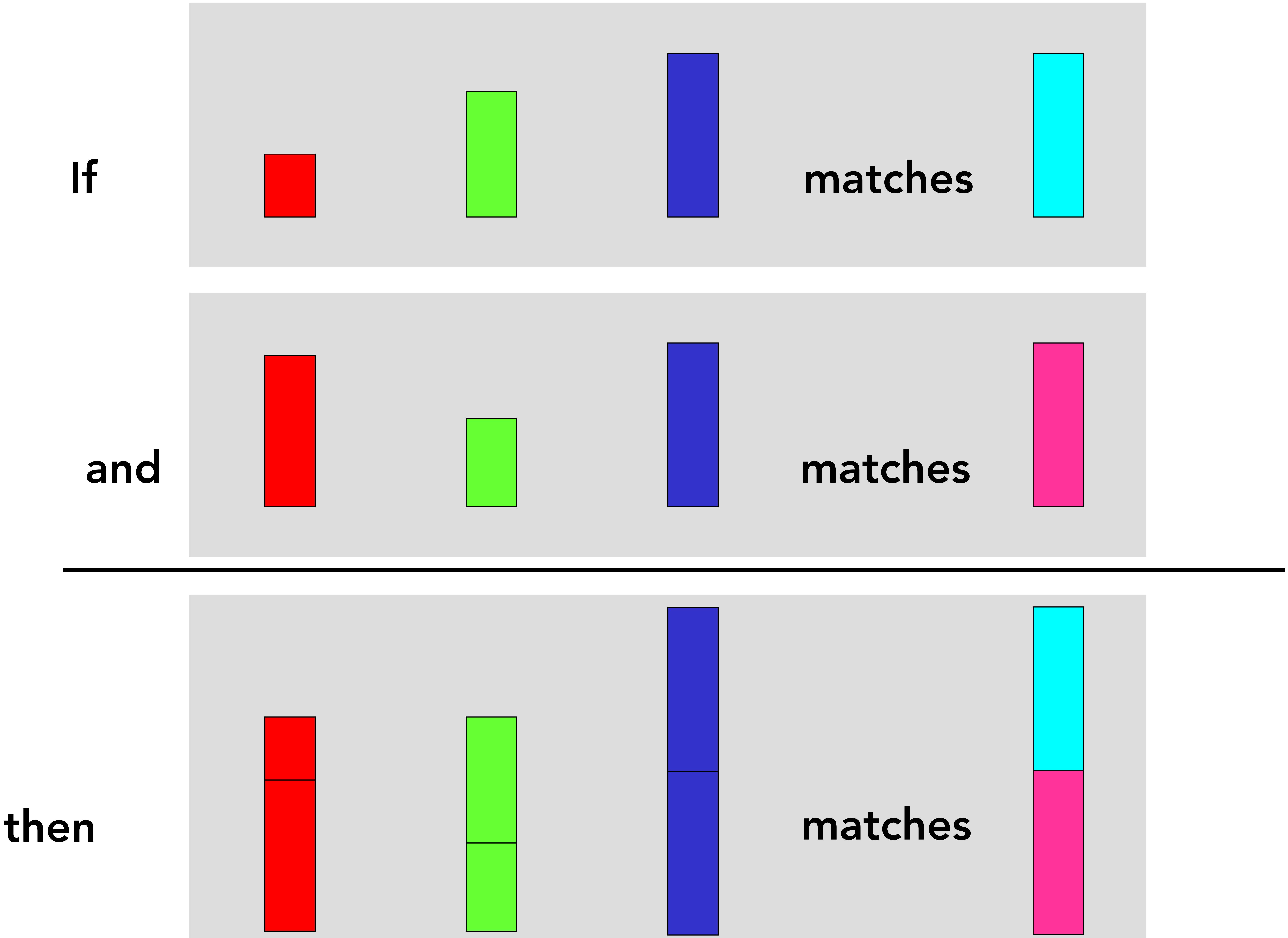
Careful: these graphs are color matching curves, not response curves or primary spectra!

Clarification

- **The previous slide plots the results of the color matching experiment for the CIE red, green, and blue laser primaries**
  - That is: the figure plots how much of <u>each primary light source</u> is needed to create a spectrum that appears as the same color as a monochromatic reference
  - Repeating the experiment for many different monochromatic light sources yields the curve in the figure
  - The color of monochromatic light is represented as a 3-vector, which can be interpreted as how much of each primary is used to make the color

- **Do not confuse these plots with the responses of S,M,L cones to monochromatic light… which was also plotted earlier in the lecture**
  - The color matching experiment works because the output of the visual system has dimensionality 3 (a.k.a. metameters that are the combination of three primaries exist), but the color matching experiment is not directly measuring the response of S,M,L cones.

The color matching experiment is linear



Brian Wandell

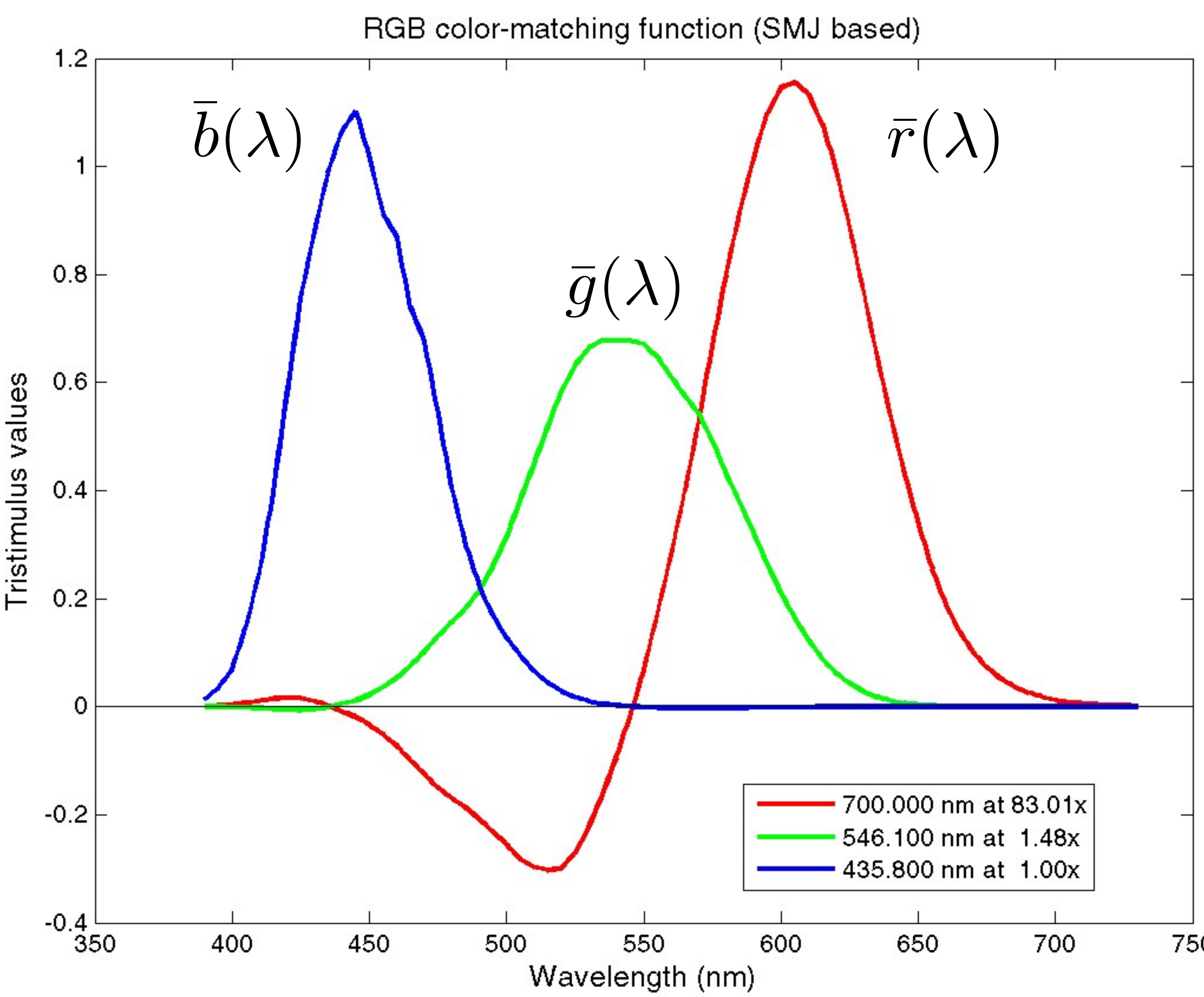
Color reproduction with matching functions

For any spectrum *s*, the perceived color is matched by the following formulas for scaling the CIE RGB primaries

$$R_{\mathrm{CIE\,RGB}} = \int_\lambda s(\lambda)\bar{r}(\lambda)\,d\lambda$$

$$G_{\mathrm{CIE\,RGB}} = \int_\lambda s(\lambda)\bar{g}(\lambda)\,d\lambda$$

$$B_{\mathrm{CIE\,RGB}} = \int_\lambda s(\lambda)\bar{b}(\lambda)\,d\lambda$$



Careful: these graphs are color matching curves: they are not response curves or primary spectra!

Color reproduction with matching functions

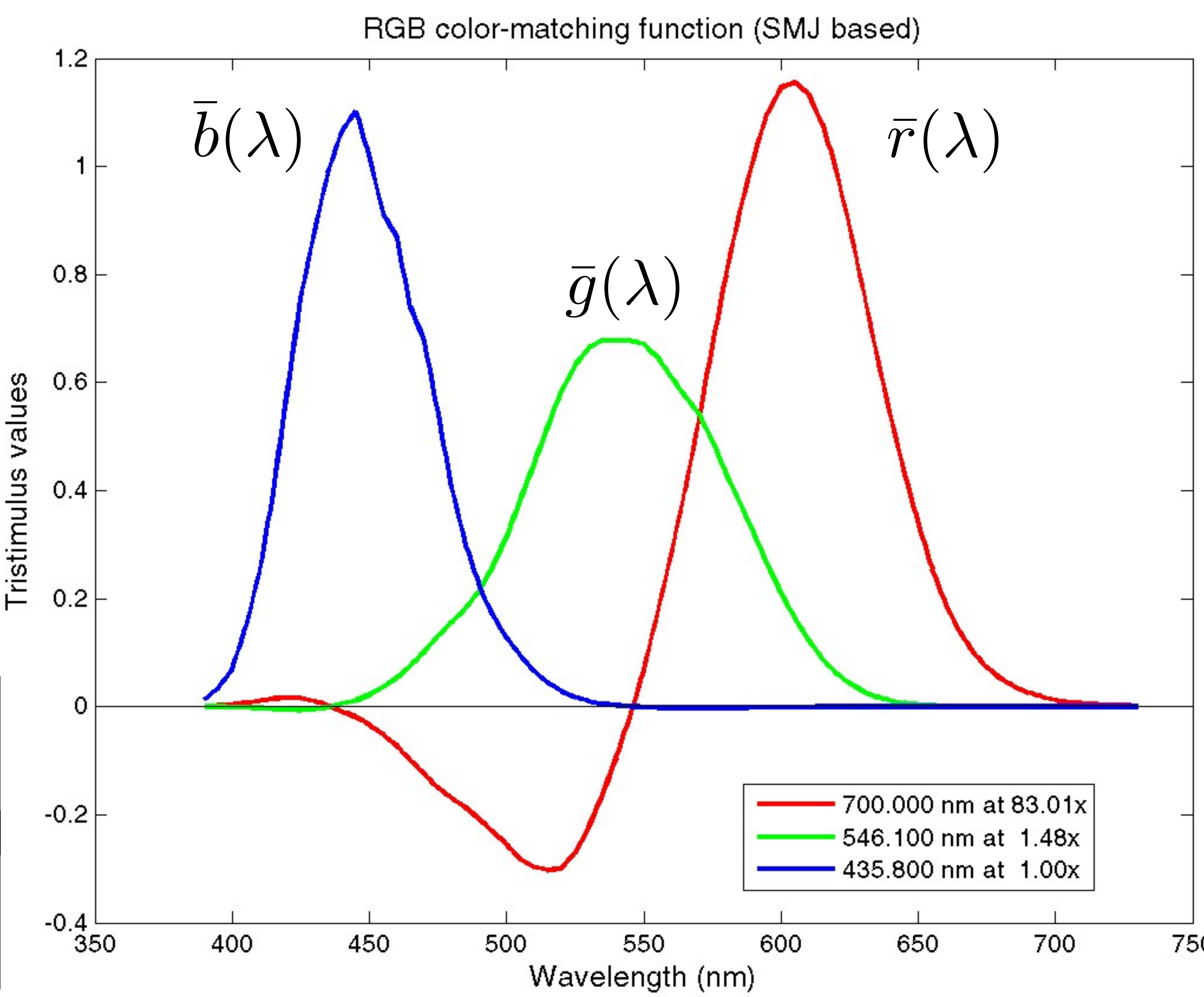
For any spectrum s, the perceived color is matched by the following formulas for scaling the CIE RGB primaries

Written as vector dot products:

$$R_{\text{CIE RGB}} = s \cdot \bar{r}$$
$$G_{\text{CIE RGB}} = s \cdot \bar{g}$$
$$B_{\text{CIE RGB}} = s \cdot \bar{b}$$

Matrix formulation:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{\text{CIE RGB}} = \begin{bmatrix} — & \bar{r} & — \\ — & \bar{g} & — \\ — & \bar{b} & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

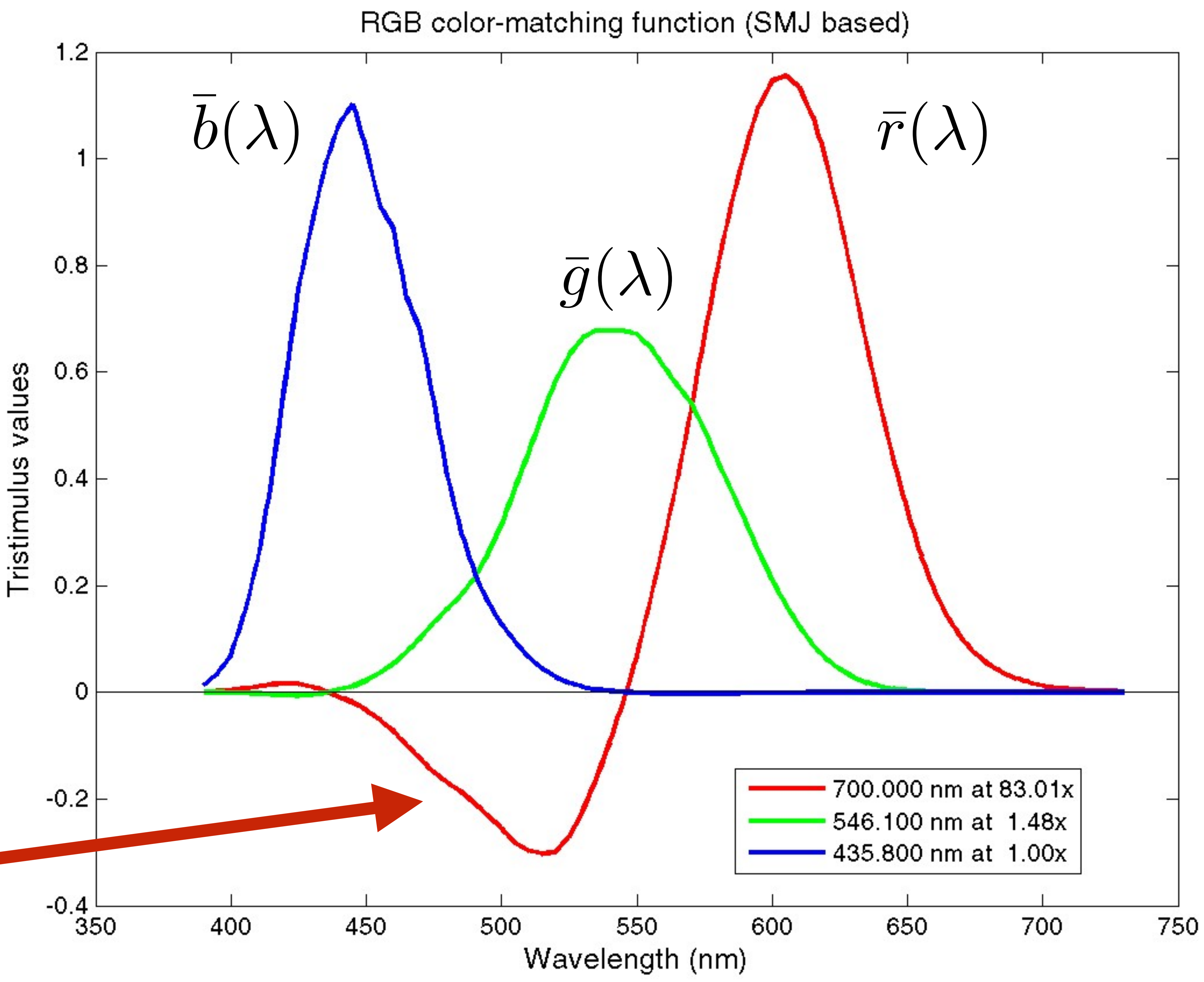


Careful: these graphs are color matching curves: they are not response curves or primary spectra!

Negative red primary?

- There is no positive combination of red, blue, green lasers that yields color that appears the same to a human as monochromatic light of 500 nm ("blue-greenish" light)

- But adding red primary to 500 nm target light yields light whose color can be matched by a combination of blue and green primaries.



Wait a minute: negative red?

Tristimulus Theory of Color

Displays producing color (additive color)

- Given a set of primary lights, each with its own spectral distribution (e.g. R,G,B display pixels):

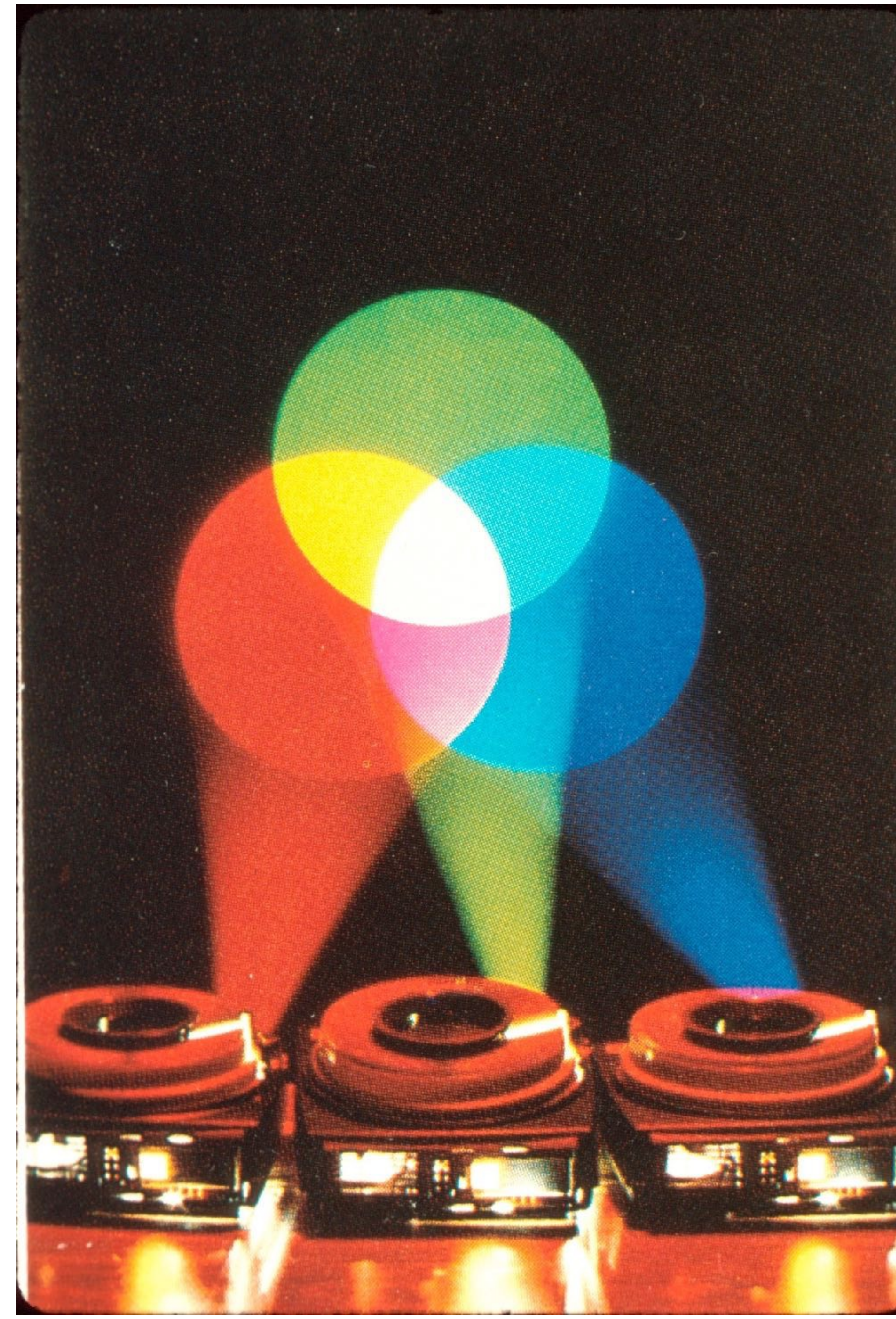
$$s_R(\lambda),\ \ s_G(\lambda),\ \ s_B(\lambda)$$

- We can adjust the brightness of these lights and add them together to produce a linear subspace of spectral distribution:

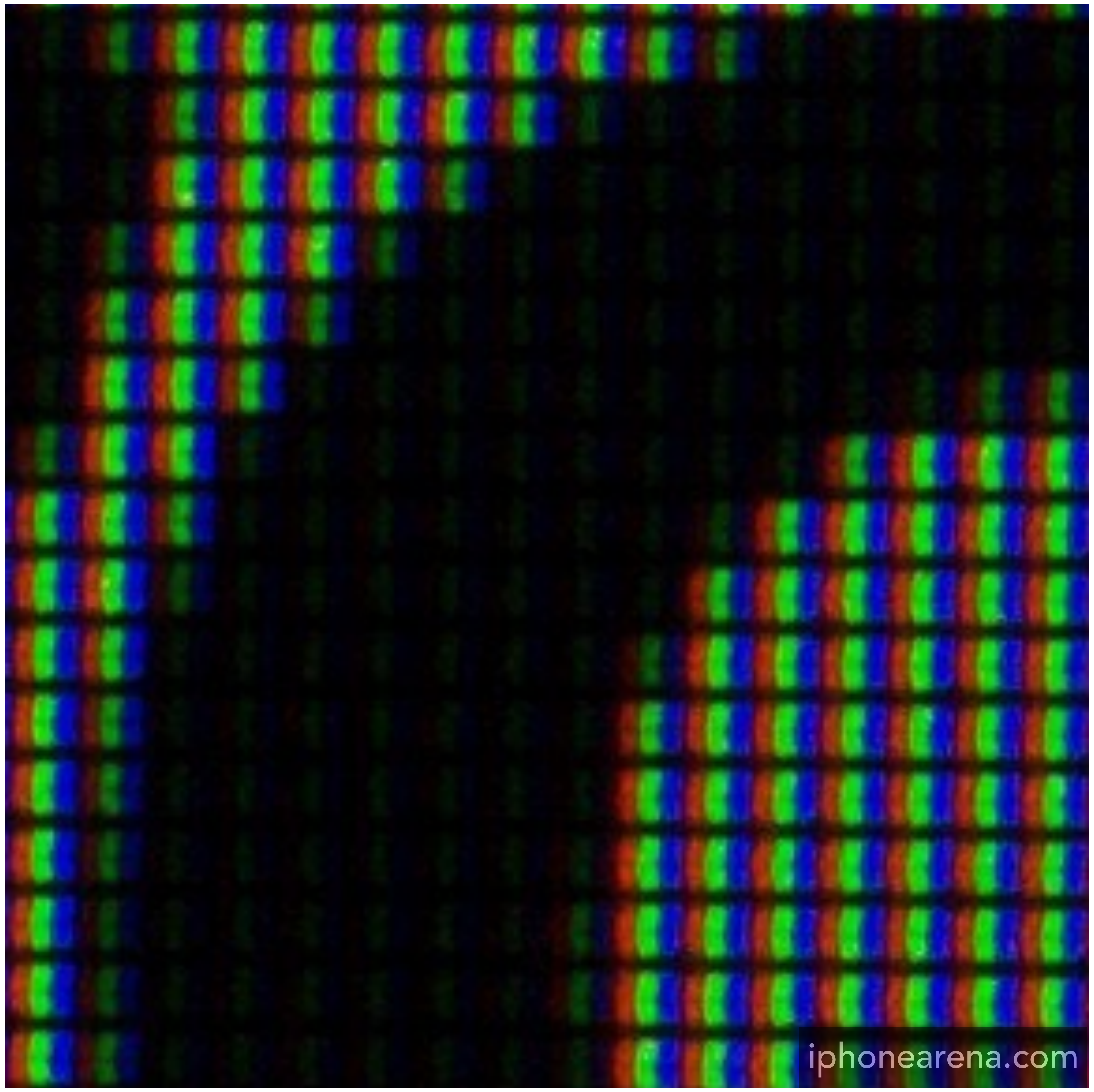
$$R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

- The color is now described by the scalar values:

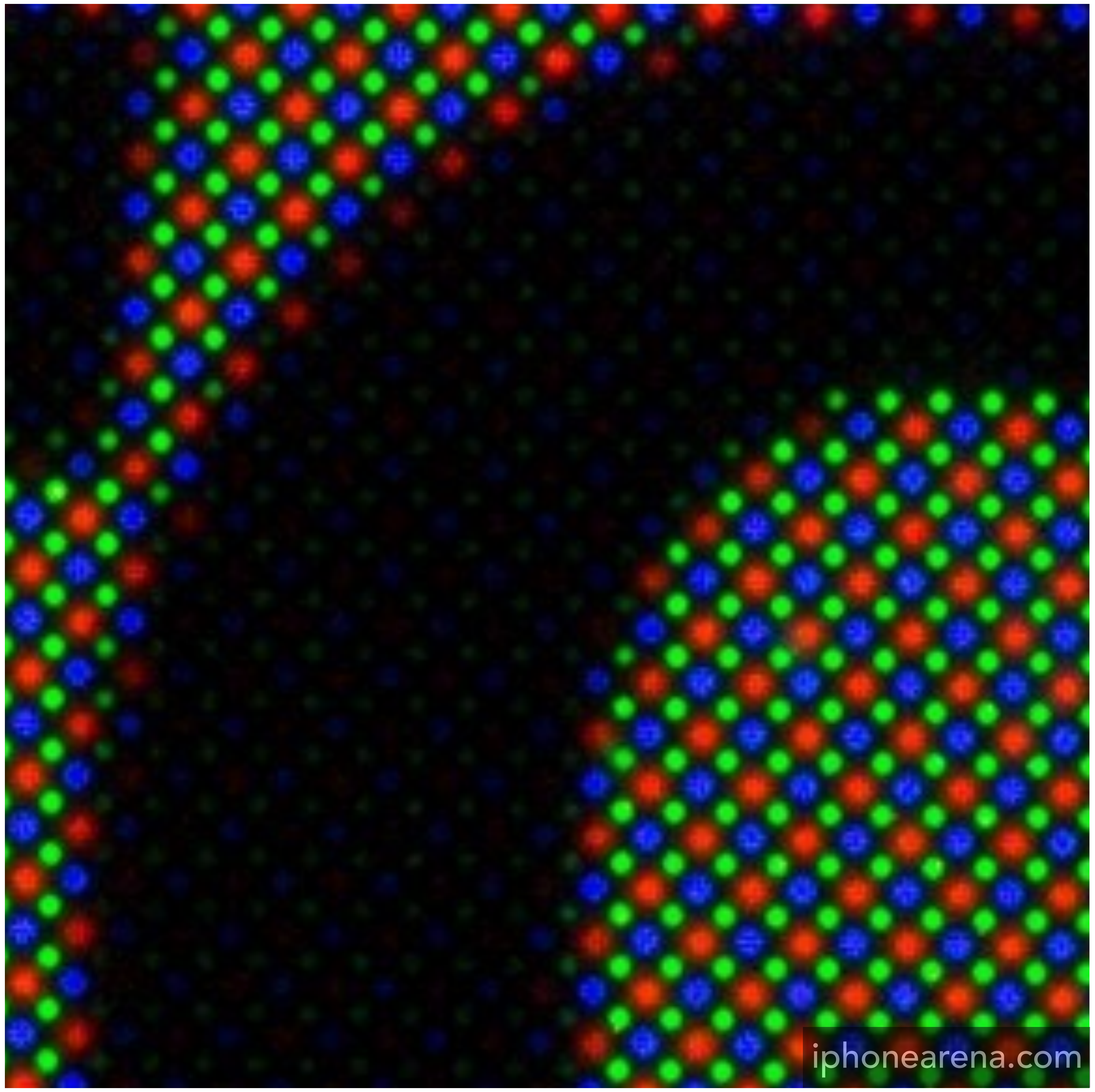
$$R,\ G,\ B$$



Recall: real LCD screen pixels (closeup)



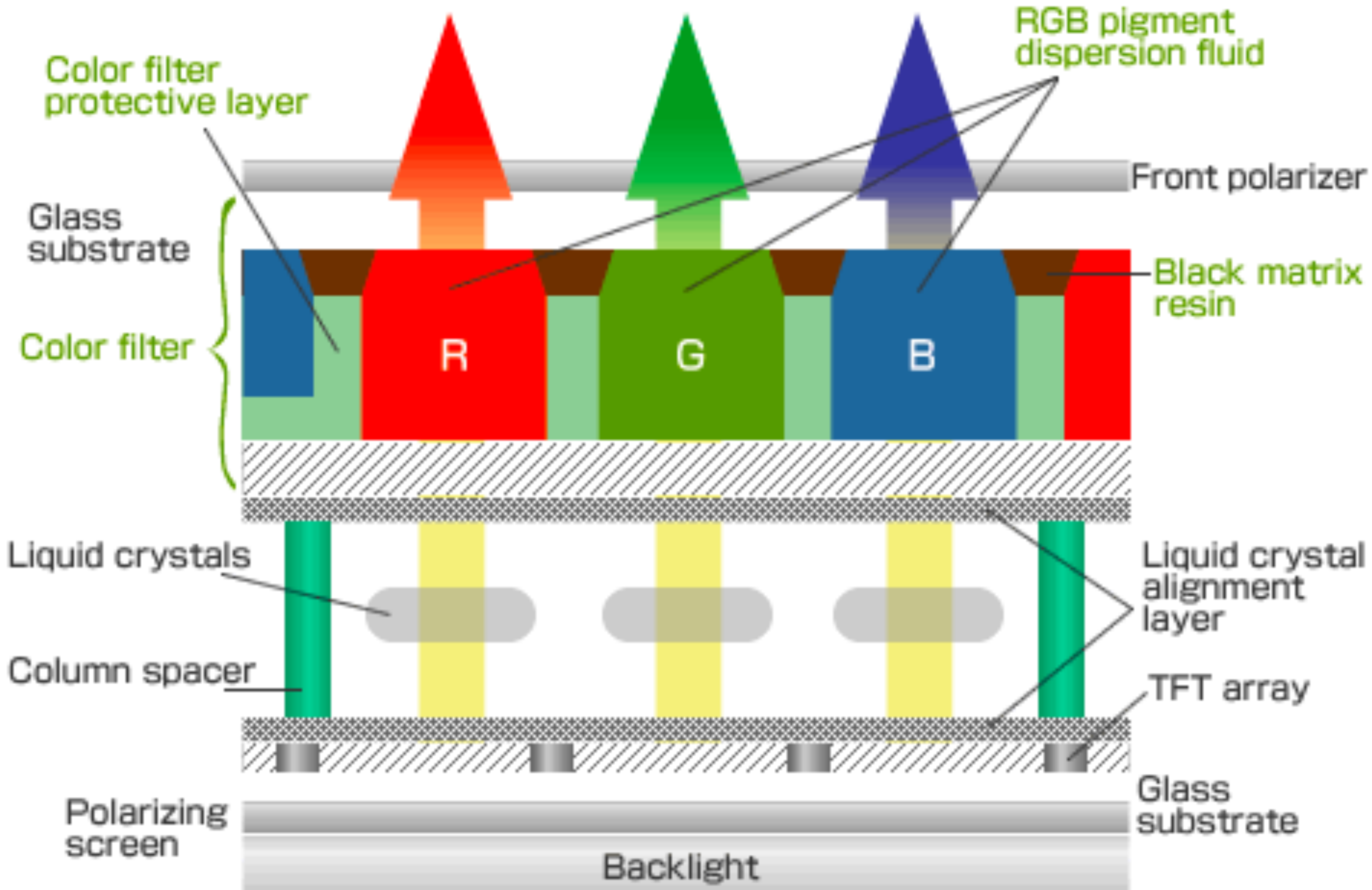
iPhone 6S



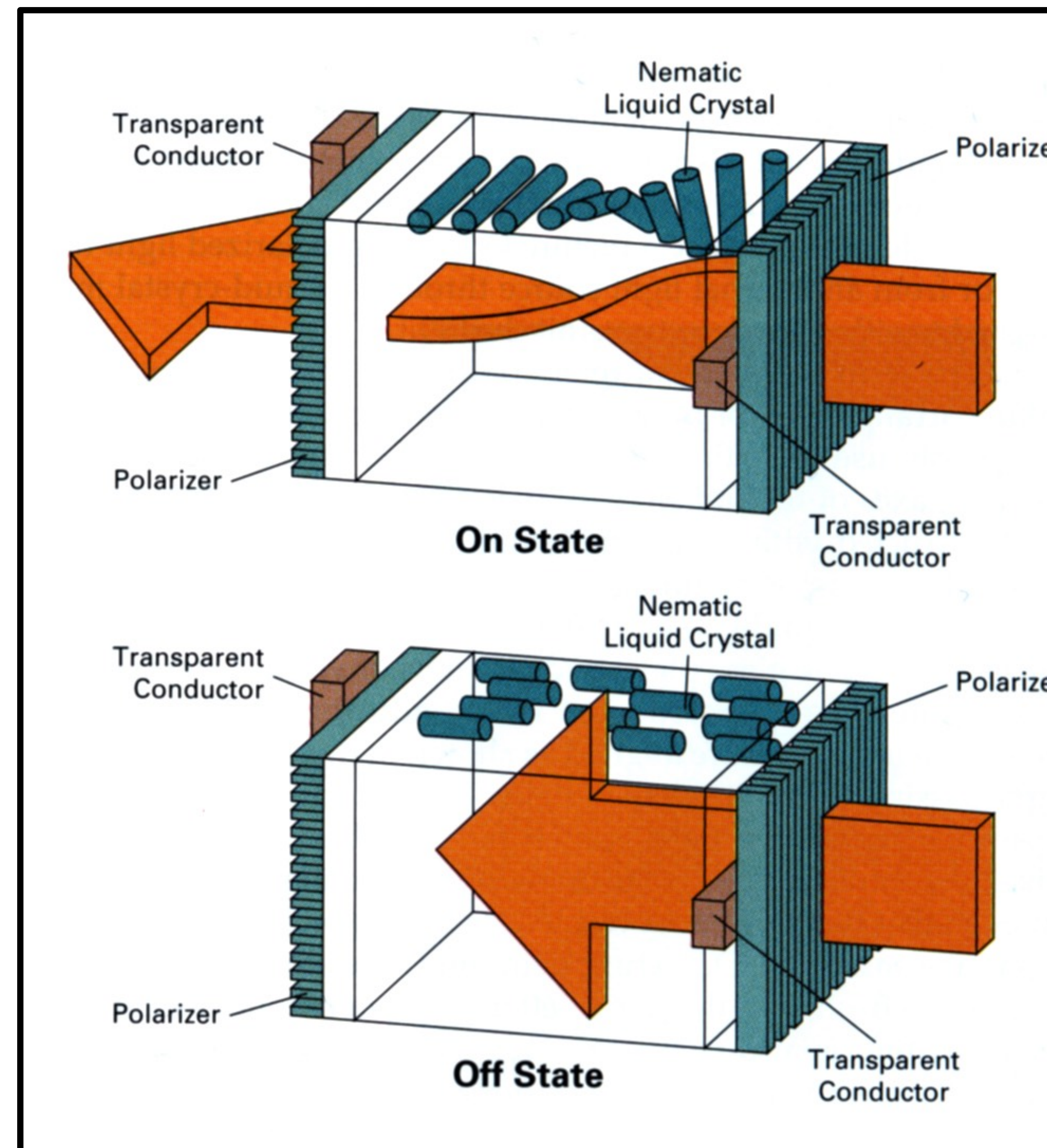
Galaxy S5

Notice R, G, B sub-pixel geometry.
Effectively three lights at each (x,y) location.

Color LCD display



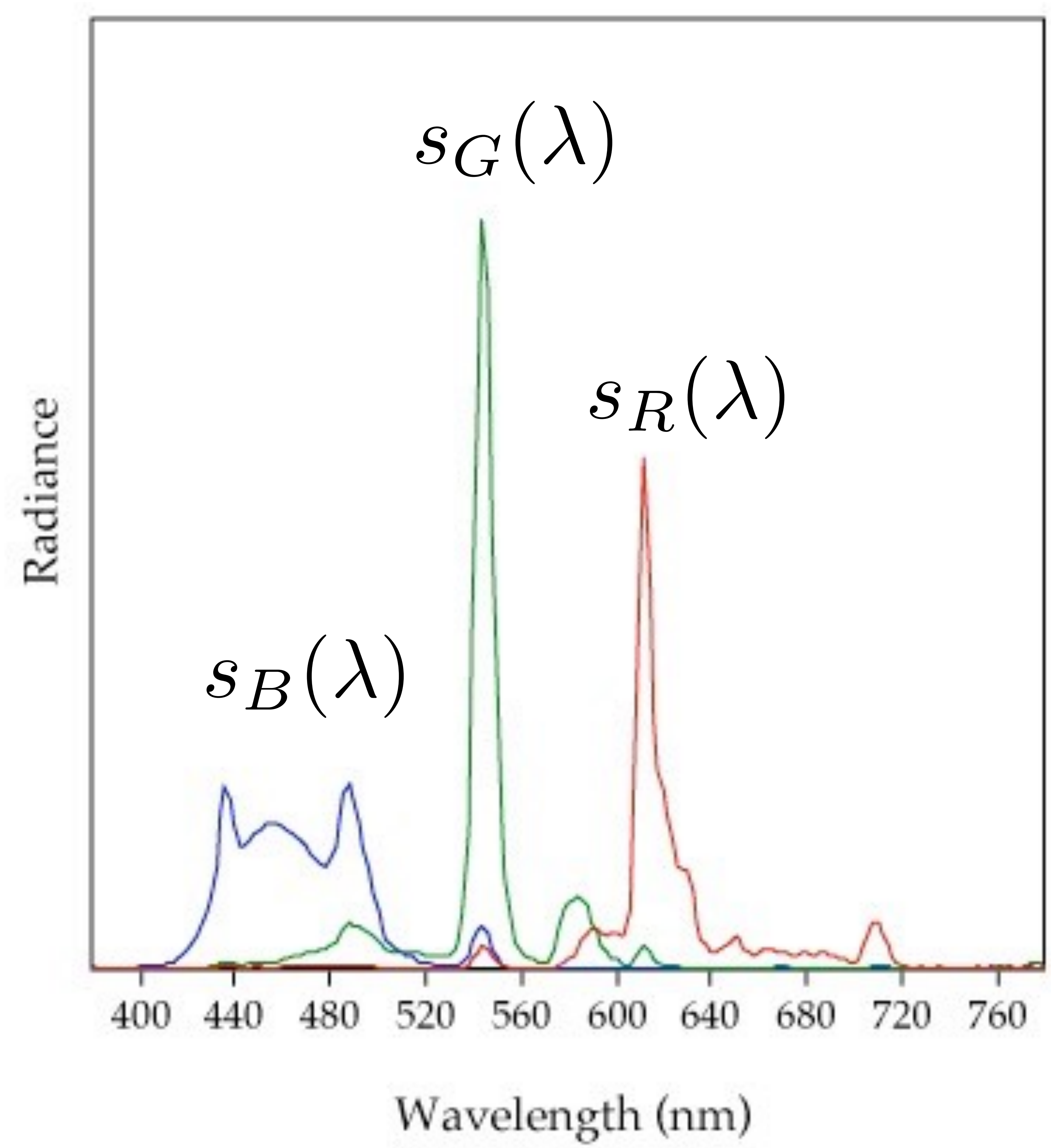
Block or transmit backlight by twisting polarization



[Image credit: H&B fig. 2-16]

[Image credit: NOF Corporation: https://www.nof.co.jp/english/business/display/product01.html]

Example primaries: LCD display



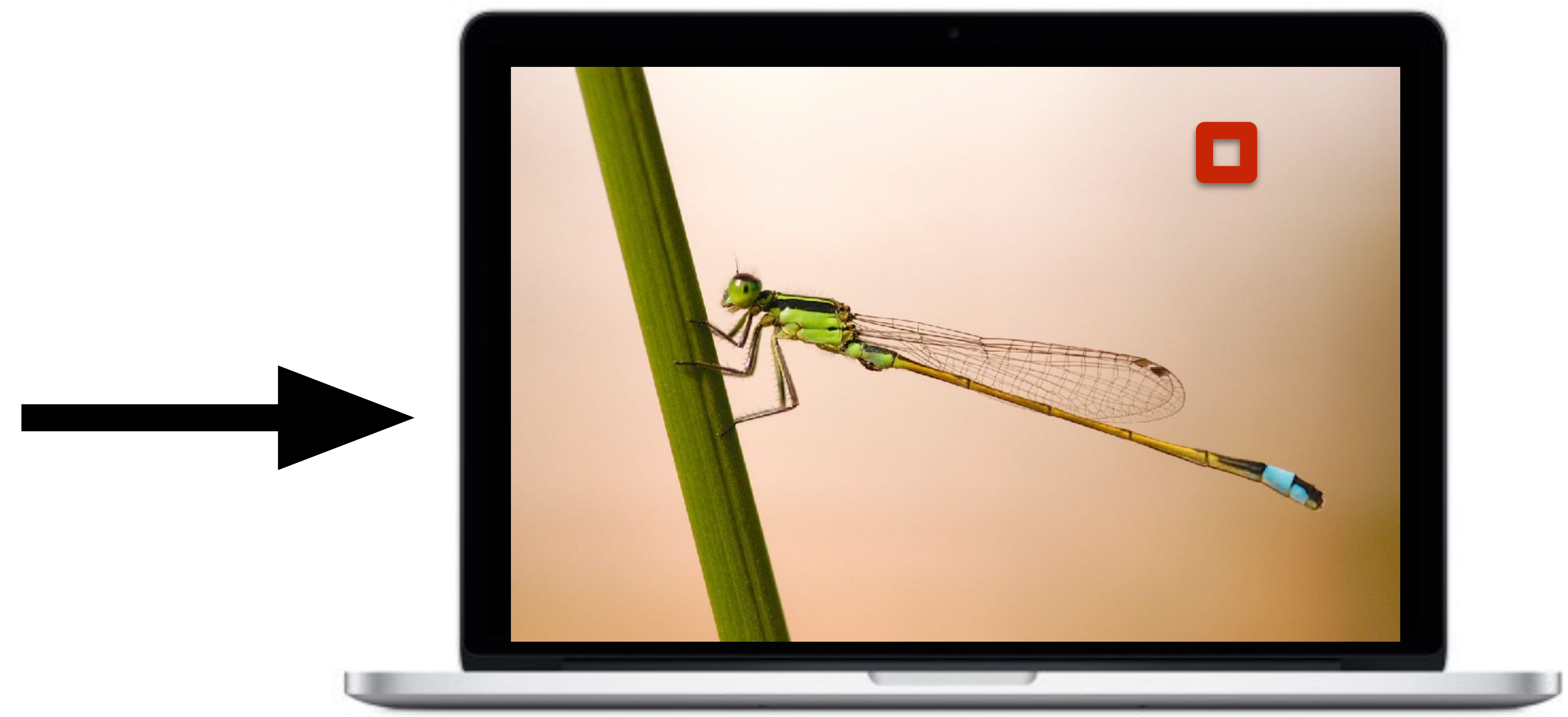
[Fairchild 97]

Spectrum of display primaries (the curves) is determined by display backlight and LCD color filters

Color reproduction problem

- Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.

Target spectrum
(what is seen in real world)



Display outputs spectrum

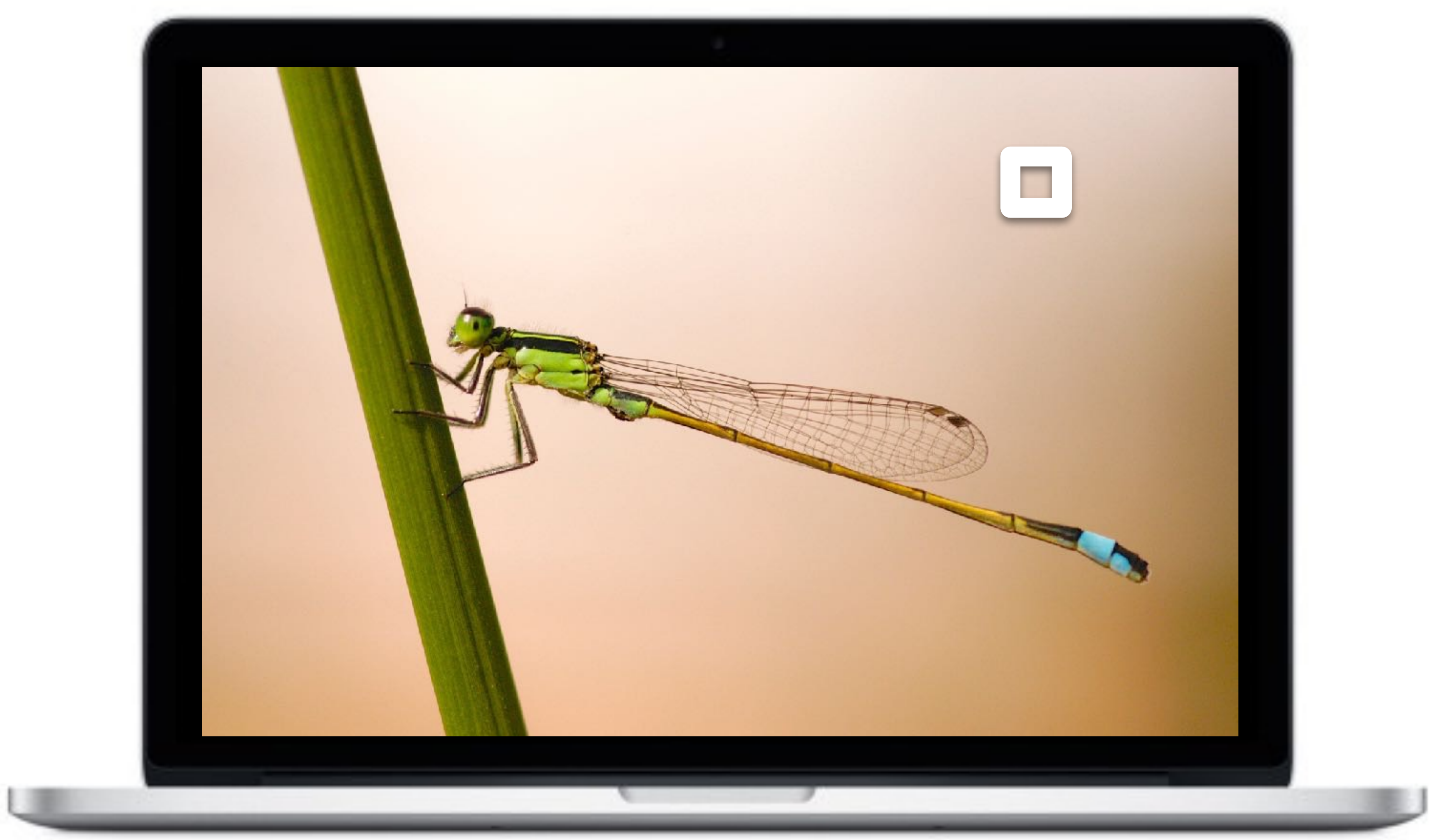
$$R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

Color reproduction as linear algebra

Spectrum produced by display given values R,G,B:

$$s_{\text{disp}}(\lambda) = R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

$$\Longrightarrow \begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



Color reproduction as linear algebra

- **What color do we perceive when we look at the display?**

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix}$$

$$= \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

We want this displayed spectrum to be a metamer for the real-world target spectrum.

Color reproduction as linear algebra

Color perceived for display spectra with values R,G,B

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color perceived for real scene spectra, s

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{real}} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

How do we reproduce the color of s?

Set these lines equal and solve for R,G,B as a function of s!

Color reproduction as linear algebra

Solution:

$$\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

$$\implies \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \left(\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \right)^{-1} \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color reproduction as linear algebra

Solution (form #1):

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \left(\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \right)^{-1} \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

1x3   Nx3   3xN   Nx3   1xN

3x3   1x3

Solution (form #2):

$$RGB = (\mathbf{M}_{SML}\,\mathbf{M}_{RGB})^{-1}\mathbf{M}_{SML}\,s$$

1x3   Nx3   3xN   Nx3   1xN

Nx3

Color reproduction as linear algebra

Solution (form #3):

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1} \underbrace{\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix}}_{\text{Nx3}} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

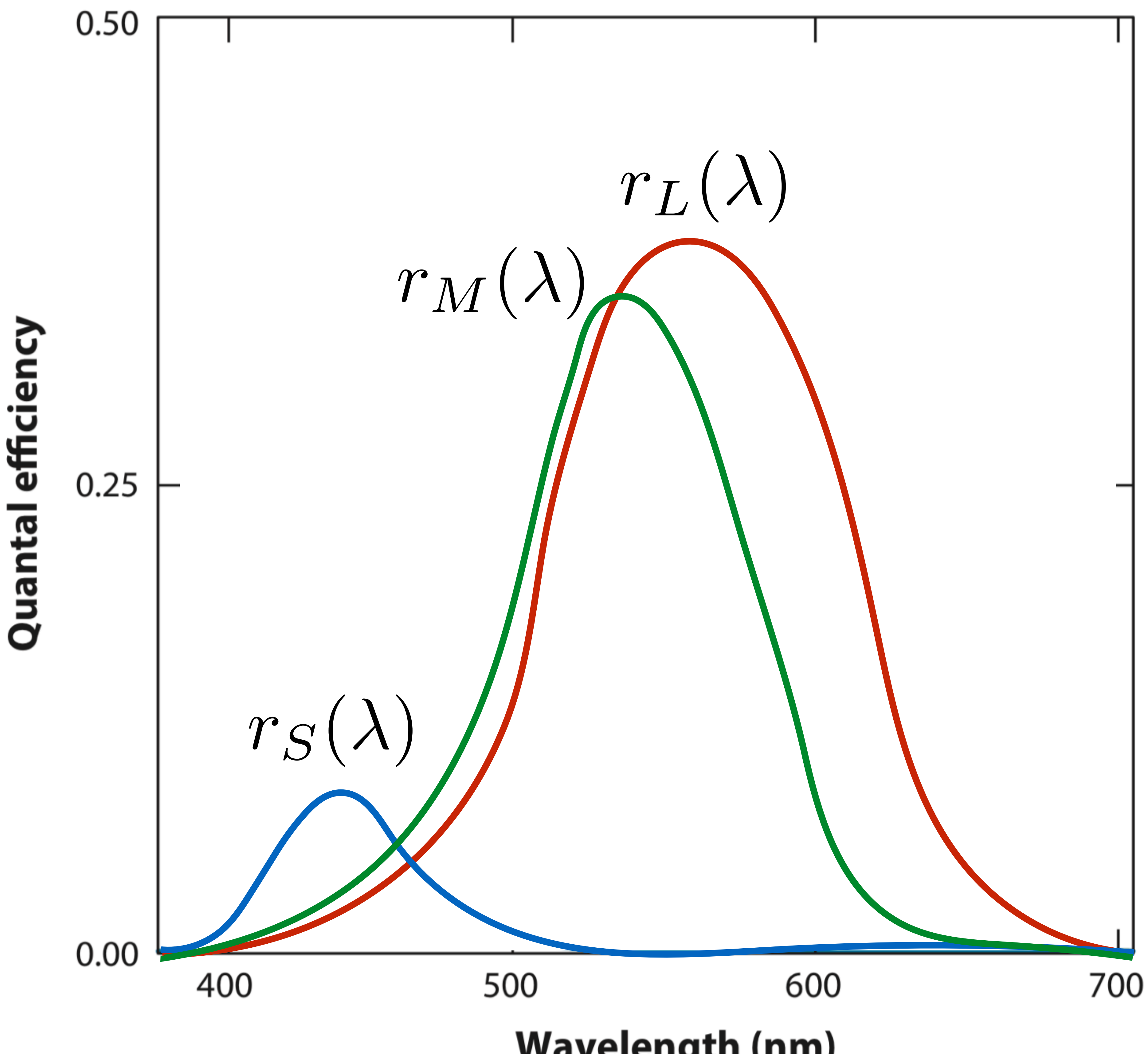
This Nx3 matrix contains, as row vectors,
"color matching functions"
associated with the primary lights s_R, s_G, s_B.

Color reproduction issue: no negative light

- R,G,B values must be positive
  - Display primaries can't emit negative light
  - But solution formulas can certainly produce negative R,G,B values
- What do negative R,G,B values mean?
  - Display can't physically reproduce the desired color
  - Desired color is outside the display's color gamut (more on this later)

Gamut

Recall: LMS response functions

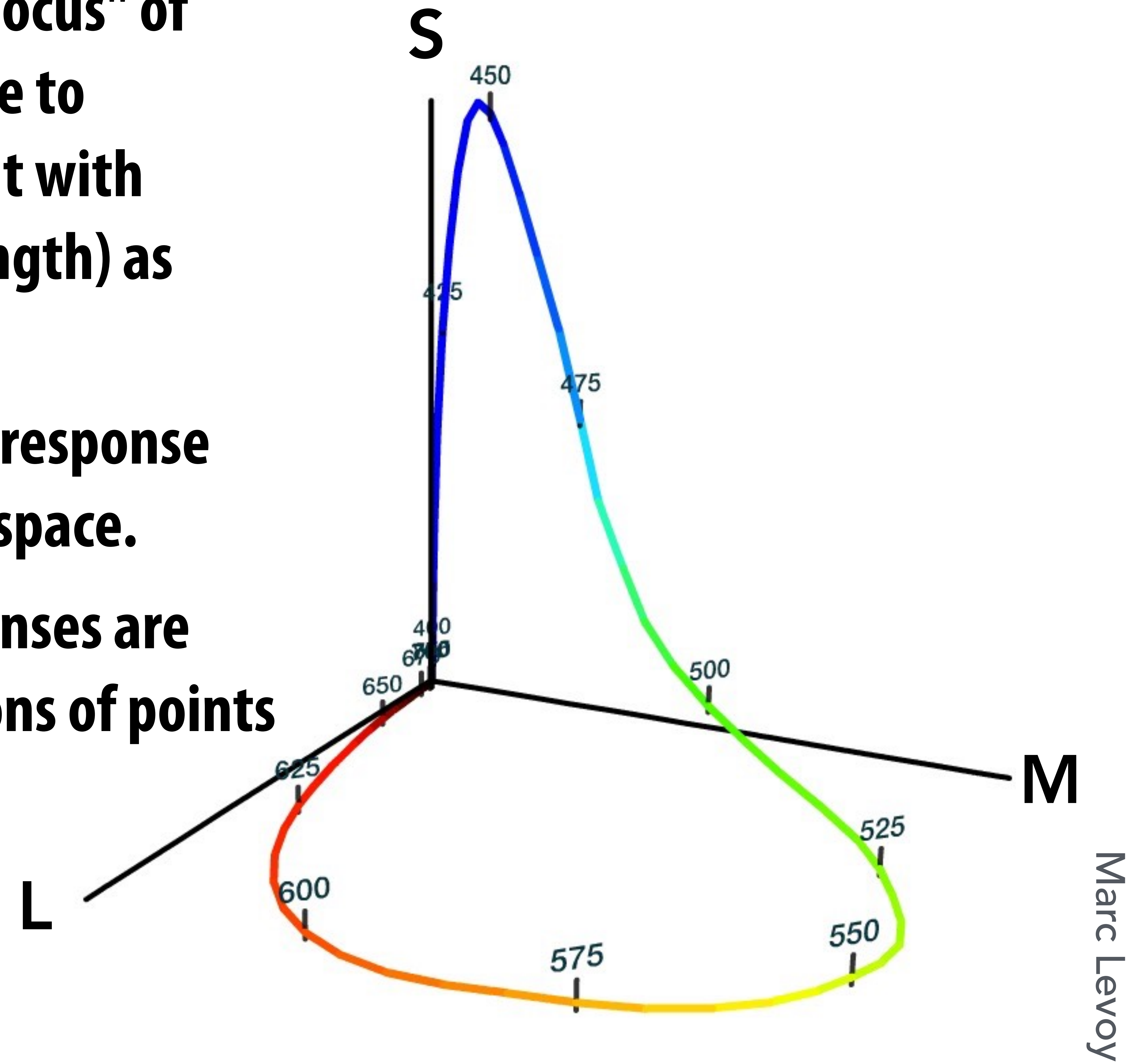


Probability that a photon will cause a photopigment isomerization

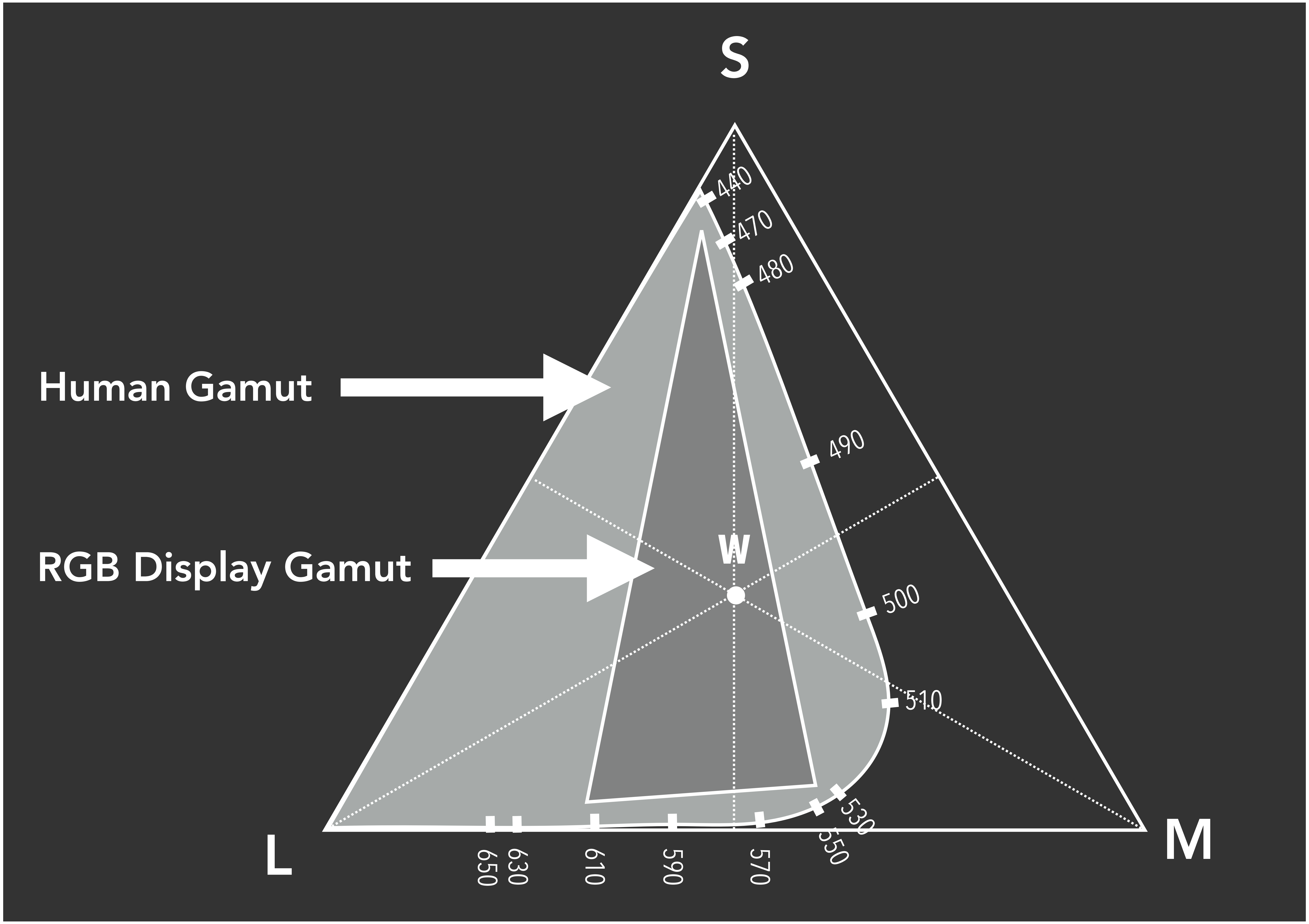
Brainard, Color and the Cone Mosaic, 2015.

LMS responses plotted as 3D color space

- Visualization of "spectral locus" of human cone cells' response to monochromatic light (light with energy in a single wavelength) as points in 3D space.
- This is a plot of the S, M, L response functions as a point in 3D space.
- Space of all possible responses are positive linear combinations of points on this curve.



Chromaticity diagram (Maxwellian)



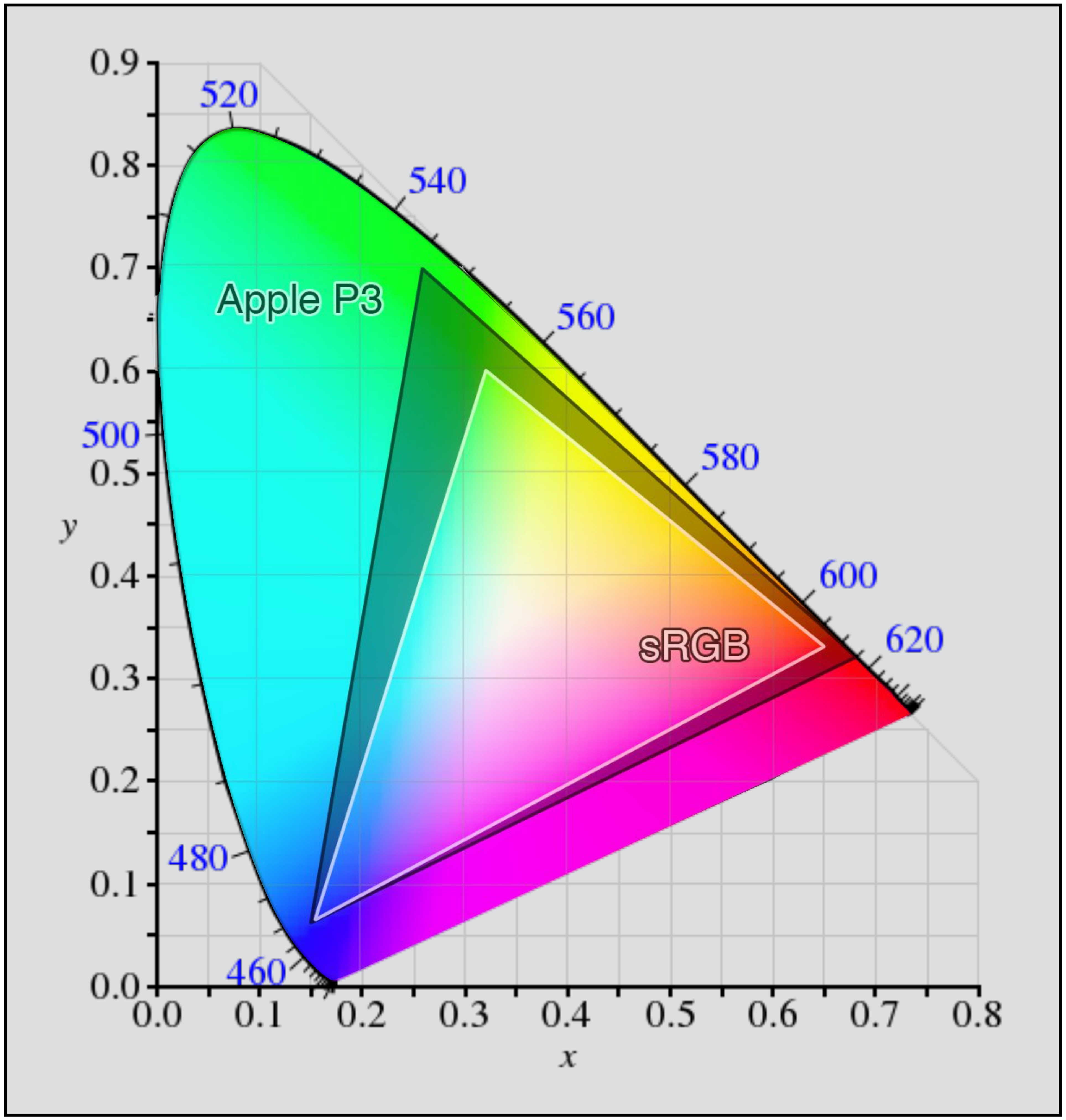
Perspective projection of spectral locus looking diagonally down at origin from (1,1,1)

Chromaticity diagram (CIE 1931 xy)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.9121 & -1.1121 & 0.2019 \\ 0.3709 & 0.6291 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

$$x = \frac{X}{|X| + |Y| + |Z|}$$

$$y = \frac{Y}{|X| + |Y| + |Z|}$$

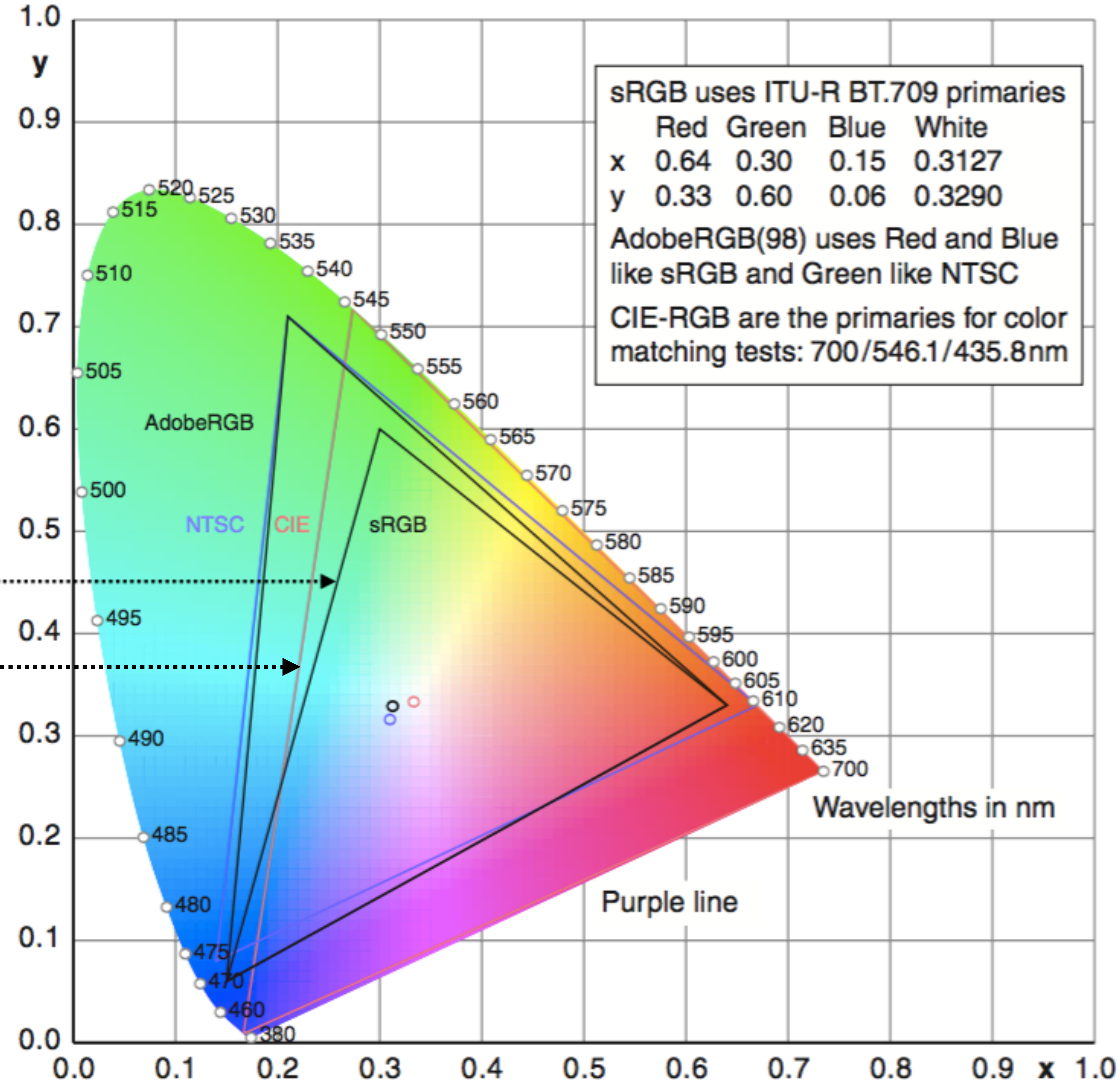


Wikipedia

Color Gamut

sRGB is a common color space used throughout the internet

CIE RGB are the monochromatic primaries used for color matching tests described earlier



Color Representation

Color Spaces

- Need three numbers to specify a color
  - But what three numbers?
  - A color space is an answer to this question
- Common example: color space defined by a display
  - Define colors by what R, G, B scalar values will produce them on your monitor
    - in math: $s = rR + gG + bB$ for some display primary spectra r, g, b
  - Device dependent: if I choose R,G,B by looking at my display and send those values to you, you may not see the same color on your display (which might have different primaries, etc.)
  - Also leaves out some colors (limited gamut), e.g. vivid yellow

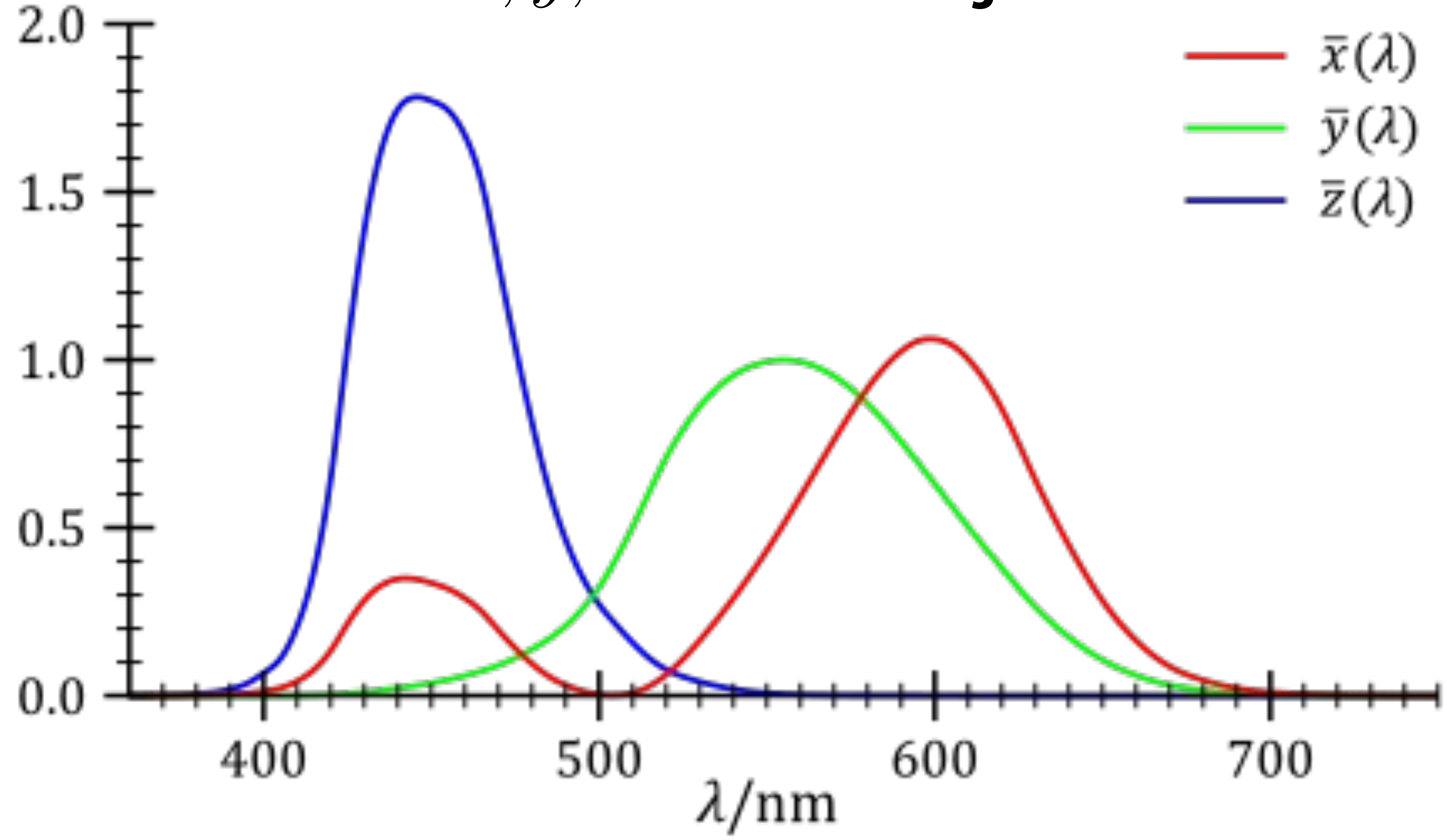
Standard color spaces

- Standardized RGB (sRGB)
  - Makes a particular monitor's primaries the RGB standard
  - Other color devices simulate that monitor by calibration
  - sRGB is usable as an interchange space; widely adopted today
  - Gamut is still limited

A "universal" color space: CIE XYZ

- Imaginary set of standard color primaries X, Y, Z
- Designed such that
  - X, Y, Z span all observable colors
  - Matching functions are strictly positive
  - Y is luminance (brightness absent color)
- "Imaginary" because the spectrum of the X,Y,Z primaries corresponding to these color matching functions are negative at some wavelengths

1931 CIE $\overline{x}, \overline{y}, \overline{z}$ color-matching functions



Careful: these graphs are color matching curves, not spectra!

For any spectrum $\Phi(\lambda)$, can express spectrum as weighted combination of primaries. Weights (X,Y,Z) given by:

$$X = k \int_\lambda \Phi(\lambda)\overline{x}(\lambda)d\lambda$$

$$Y = k \int_\lambda \Phi(\lambda)\overline{y}(\lambda)d\lambda$$

$$Z = k \int_\lambda \Phi(\lambda)\overline{z}(\lambda)d\lambda$$

[Image credit: Wikipedia]

Mathematically: just a change of basis

- By definition, all observable monochromatic spectra are positive points in XYZ space, so can convert a color's representation (in space defined by realizable primaries like RGB) to XYZ via a linear transform:
  - Consider display with 3 primaries (primaries need not be monochromatic light)
  - Compute XYZ coords of light emitted by display when providing it (1,0,0), (0,1,0), (0,0,1)
  - Light generated by display is linear combination of these vectors (non-negative weights)

color of R primary ([1,0,0] on display) $= R_x\mathbf{X} + R_y\mathbf{Y} + R_z\mathbf{Z}$

color of G primary ([0,1,0] on display) $= G_x\mathbf{X} + G_y\mathbf{Y} + G_z\mathbf{Z}$

color of B primary ([0,1,0] on display) $= B_x\mathbf{X} + B_y\mathbf{Y} + B_z\mathbf{Z}$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} R_x & G_x & B_x \\ R_y & G_y & B_y \\ R_z & G_z & B_z \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

XYZ representation

color in space of display primaries

- Example: Converting from CIE RGB to CIE XYZ:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17687 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Brightness

- The color matching experiments measure how a human observer perceives color. The goal was to match the perceived color of one spectrum with a new spectrum (a metamer) formed via the combination of three primaries.

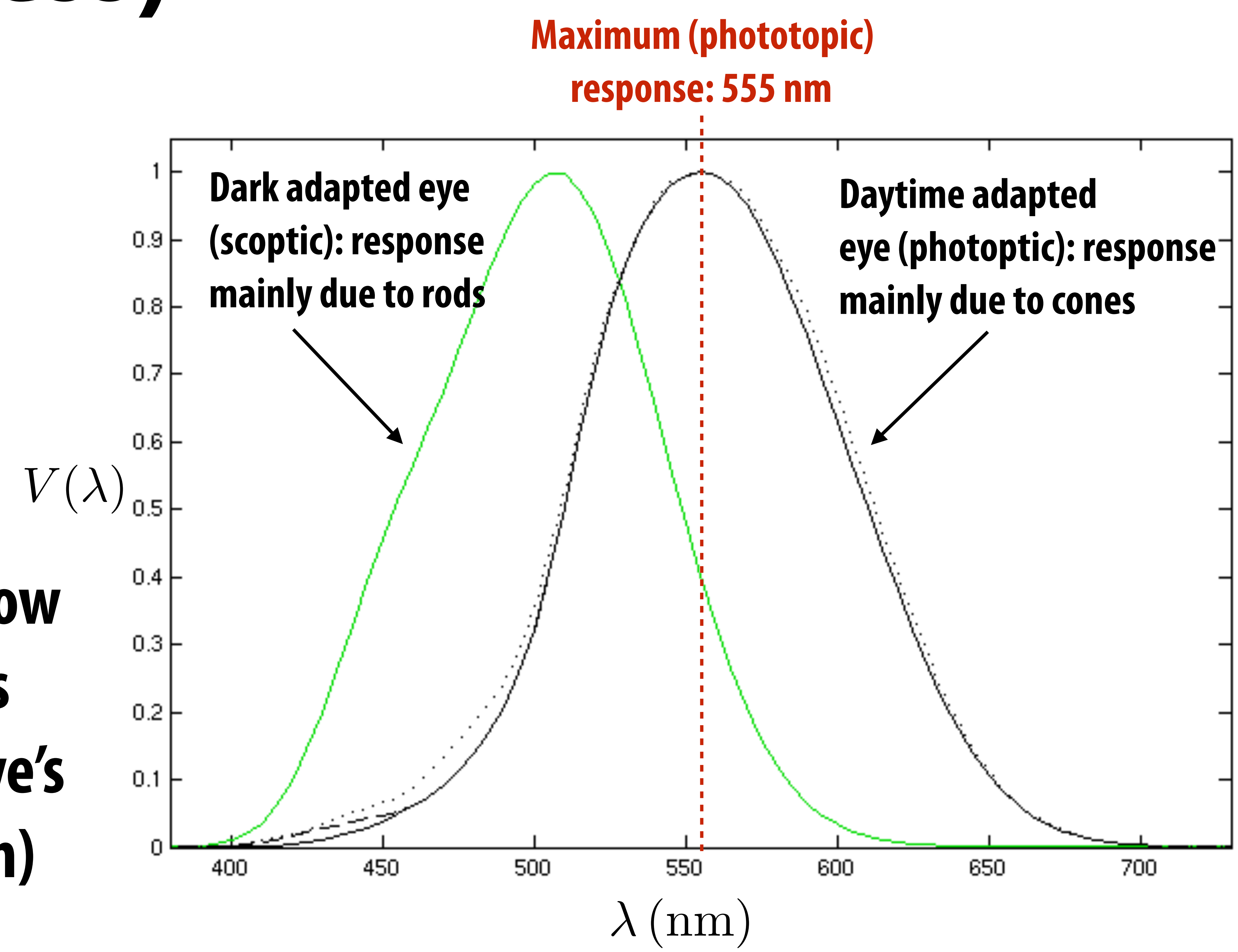
- We can also ask the question, given lights with two different colors but equal power, how bright do the lights look?

Luminance (brightness)

- **Product of radiance and the eye's luminous efficiency**

$$Y = \int \Phi(\lambda) V(\lambda) \, \mathrm{d}\lambda$$

- **Luminous efficiency is measure of how bright light at a given wavelength is perceived by a human (due to the eye's response to light at that wavelength)**



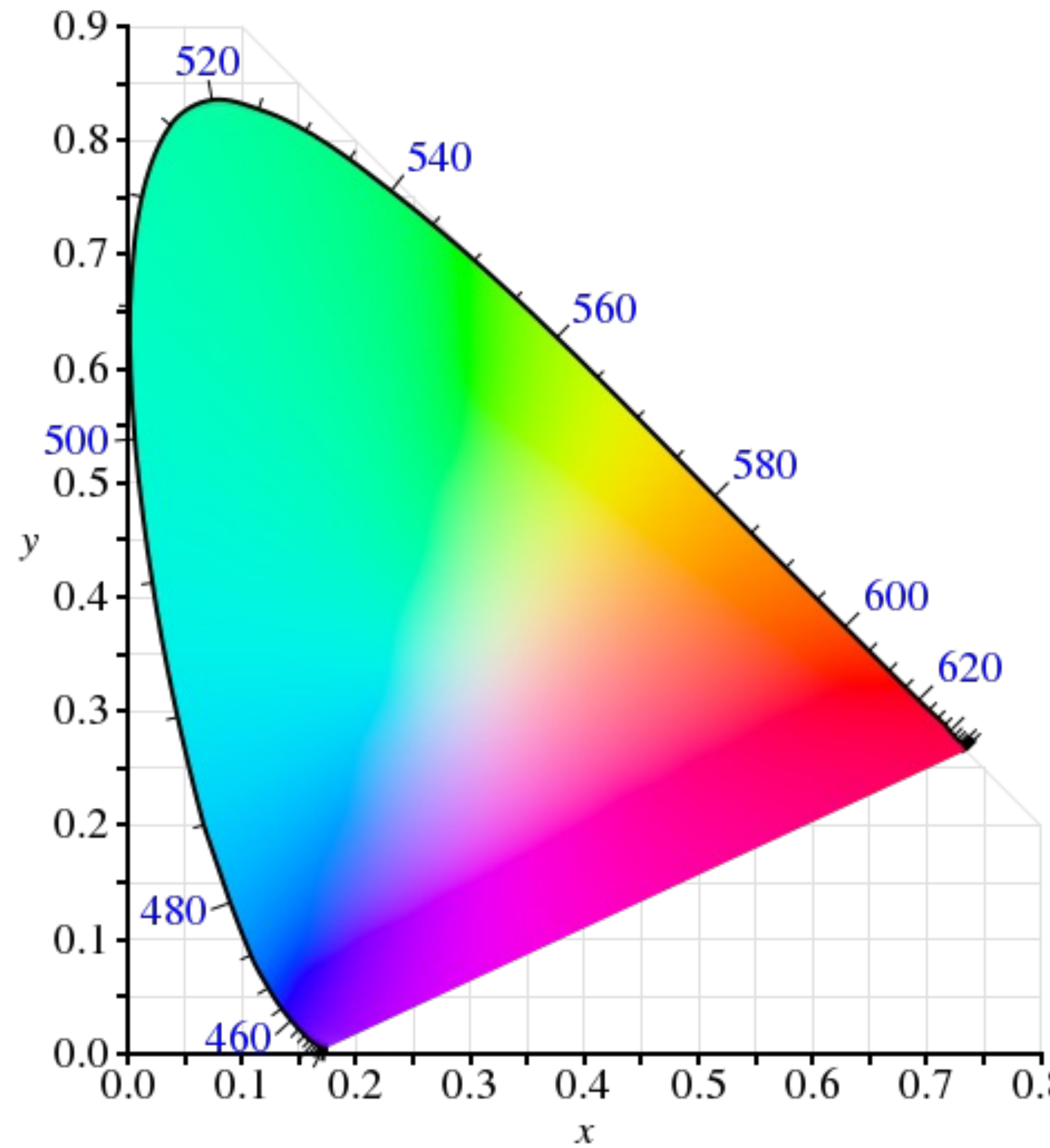
- **How to measure the eye's response curve $V(\lambda)$?**
  - **Adjust power of monochromatic light source of wavelength λ until it matches the brightness of reference 555 nm source (photopic case)**
  - **Notice: the sensitivity of photopic eye is maximized at ~ 555 nm**

https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png

Separating Luminance, Chromaticity

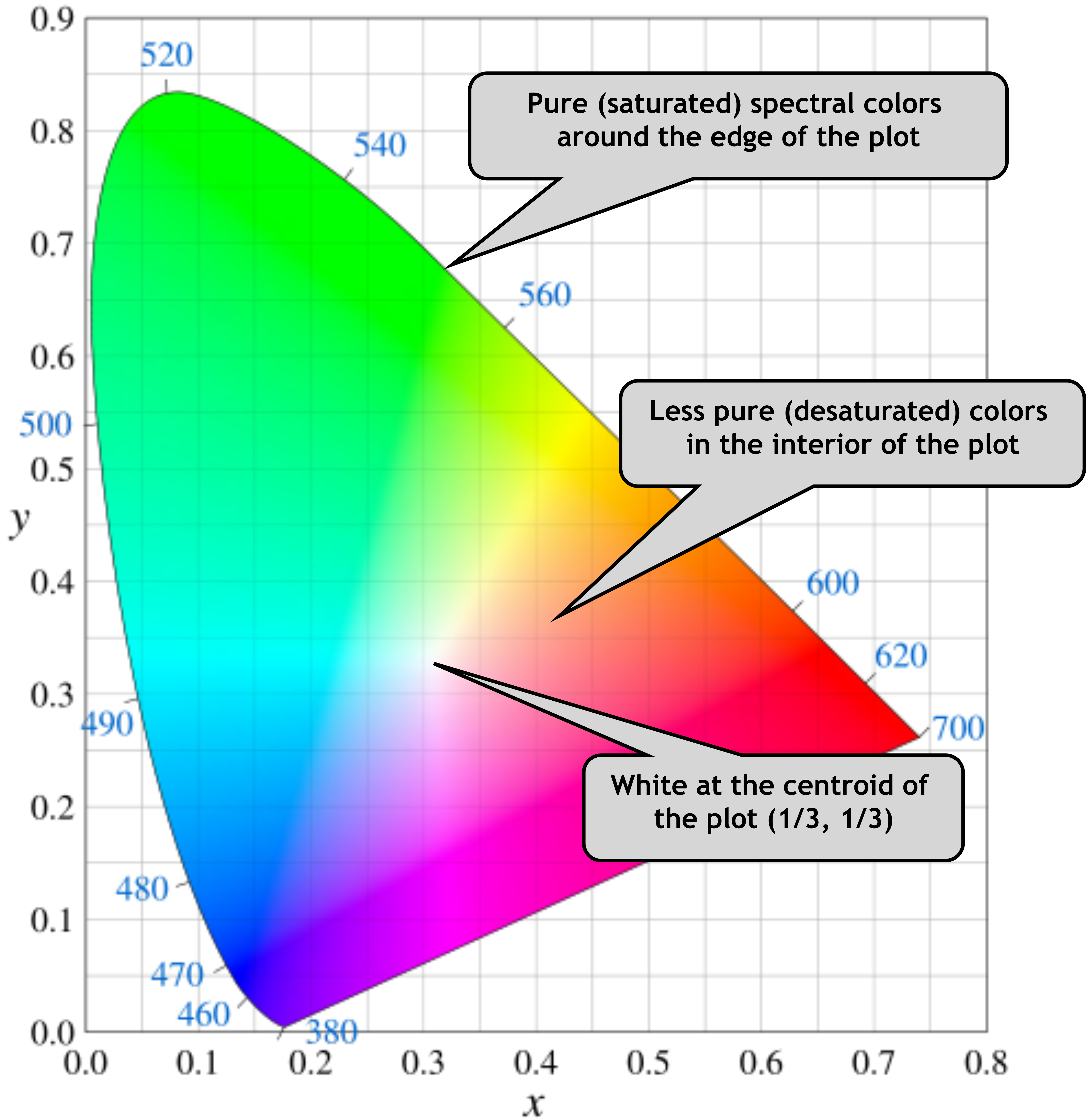
- **Luminance: *Y***
- **Chromaticity: *x, y, z*, defined as**

$$x = \frac{X}{X+Y+Z}$$
$$y = \frac{Y}{X+Y+Z}$$
$$z = \frac{Z}{X+Y+Z}$$



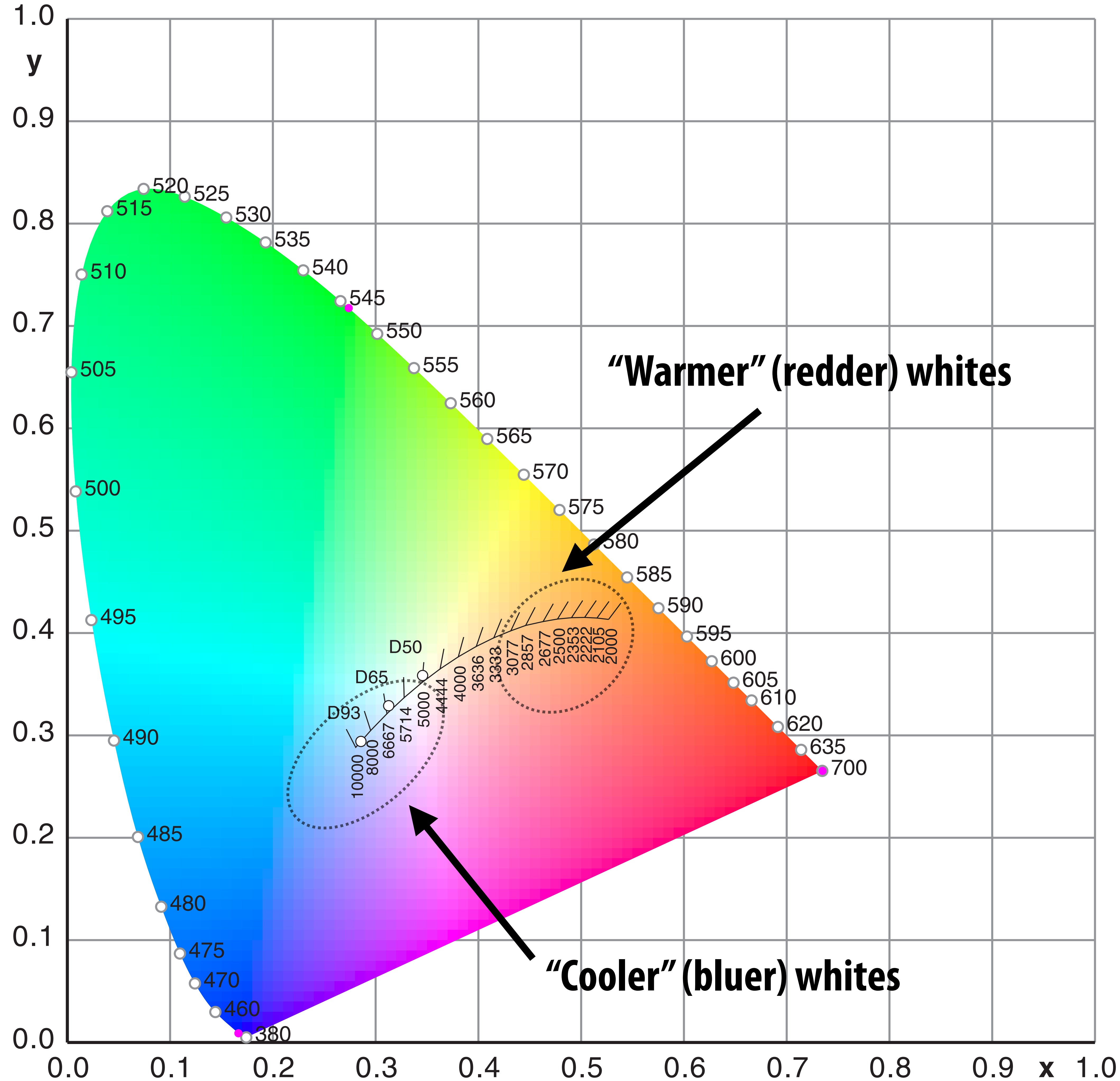
  - since $x + y + z = 1$, we only need to record two of the three
  - usually choose x and y, leading to (x, y, Y) coords

CIE chromaticity diagram



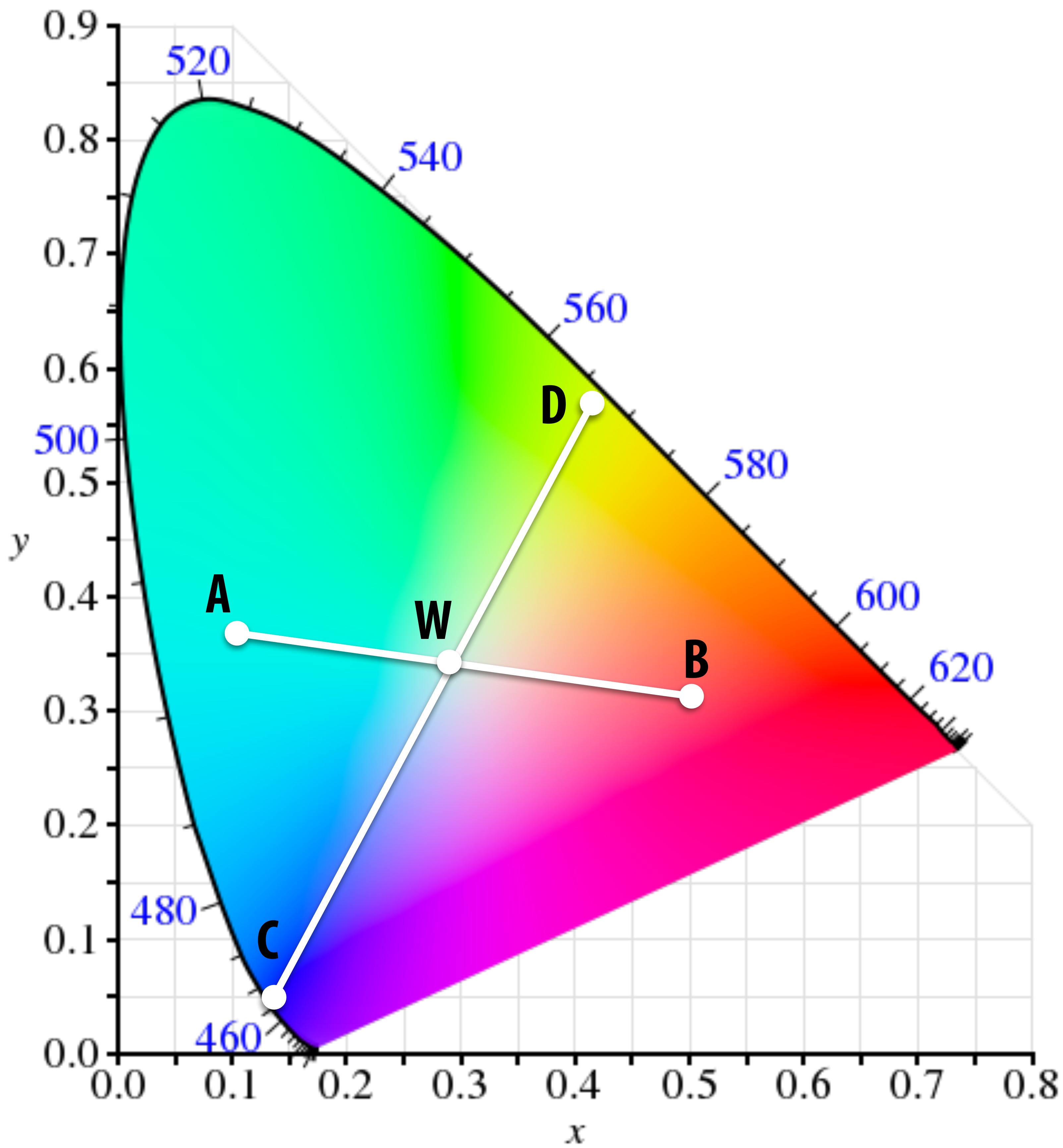
What is white?

“White point” of a display is the X,Y,Z color space vale of the point (1,1,1) in the color space defined by the display’s primaries



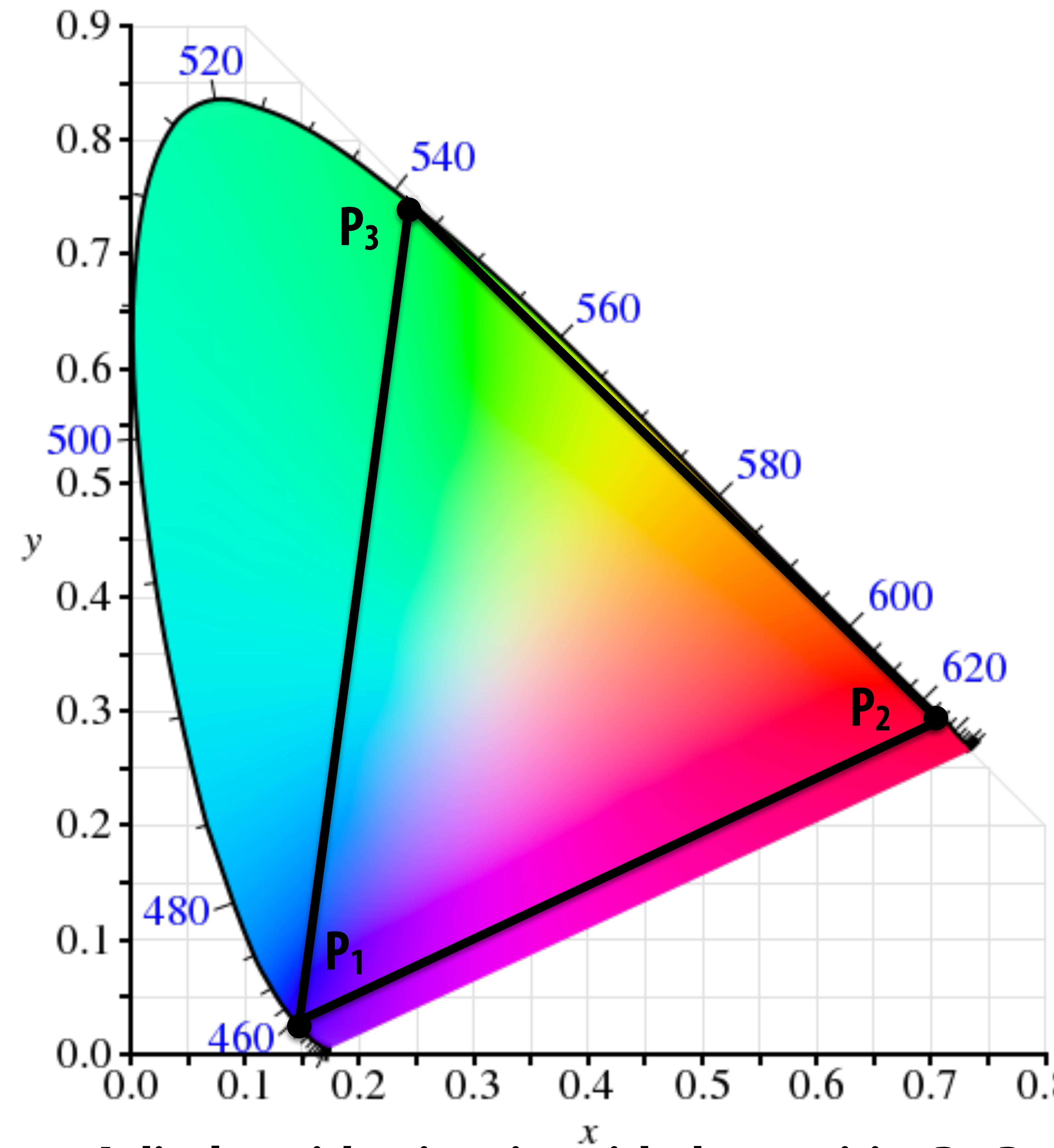
Uses of chromaticity diagram

Complementary colors are colors that can be mixed to form a designated white.



A and B and C and D are complementary with respect to reference white W

Demonstrate colors that fall out-of-gamut for a given choice of primaries

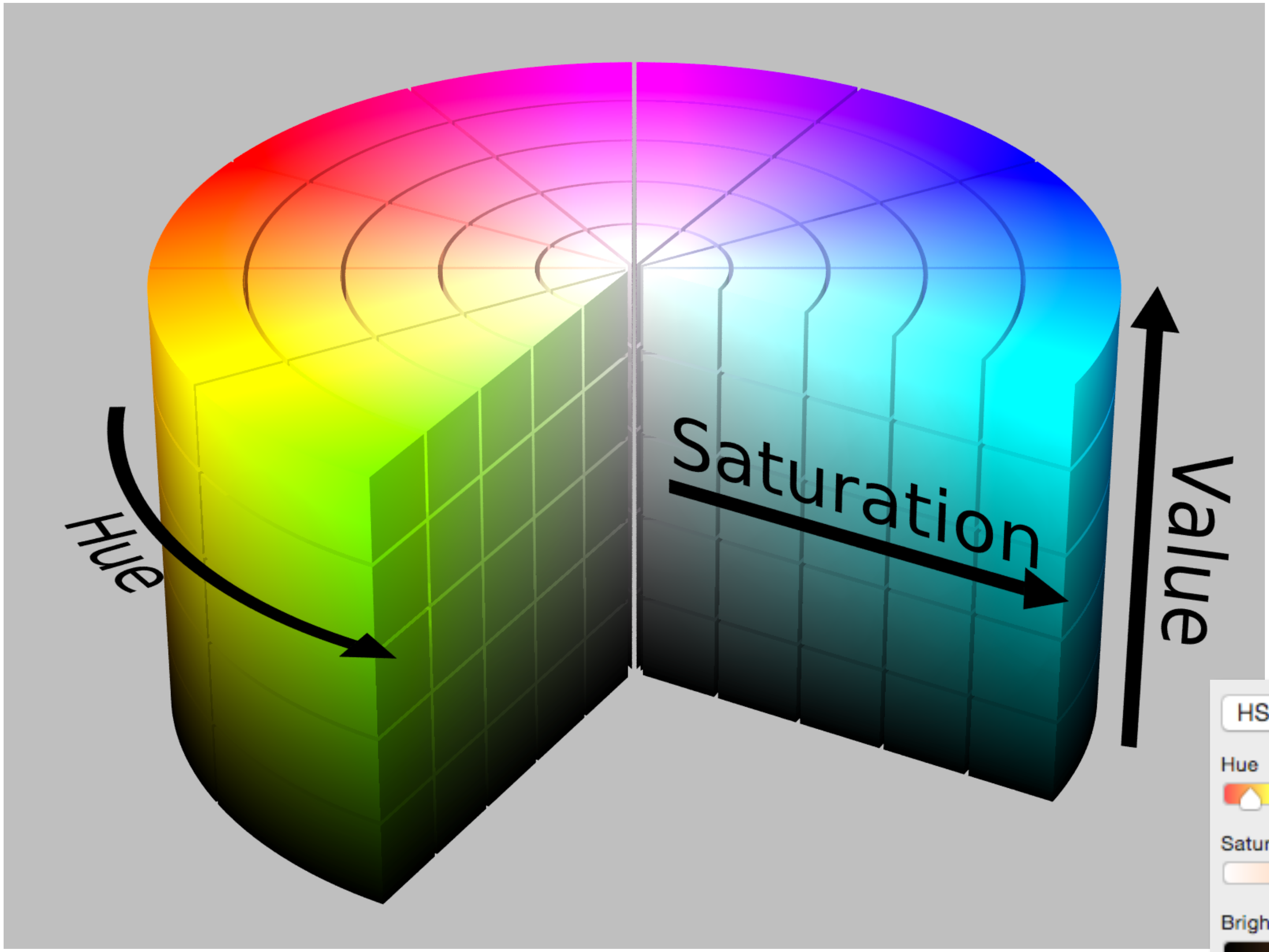


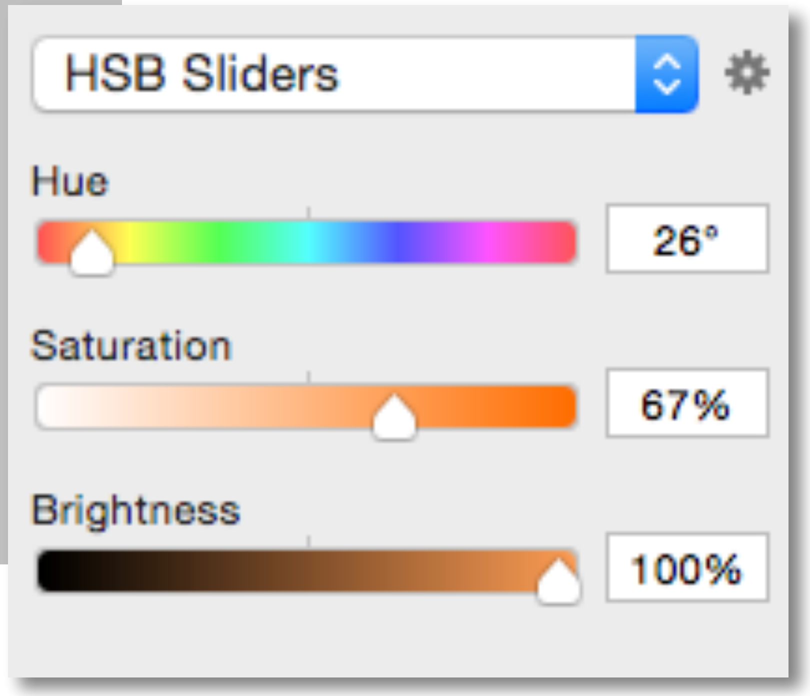
A display with primaries with chromacities P_1, P_2, P_3 can create colors that are combinations of these primaries (colors that fall within the triangle)

Perceptually Organized Color Spaces

HSV (hue-saturation-value)

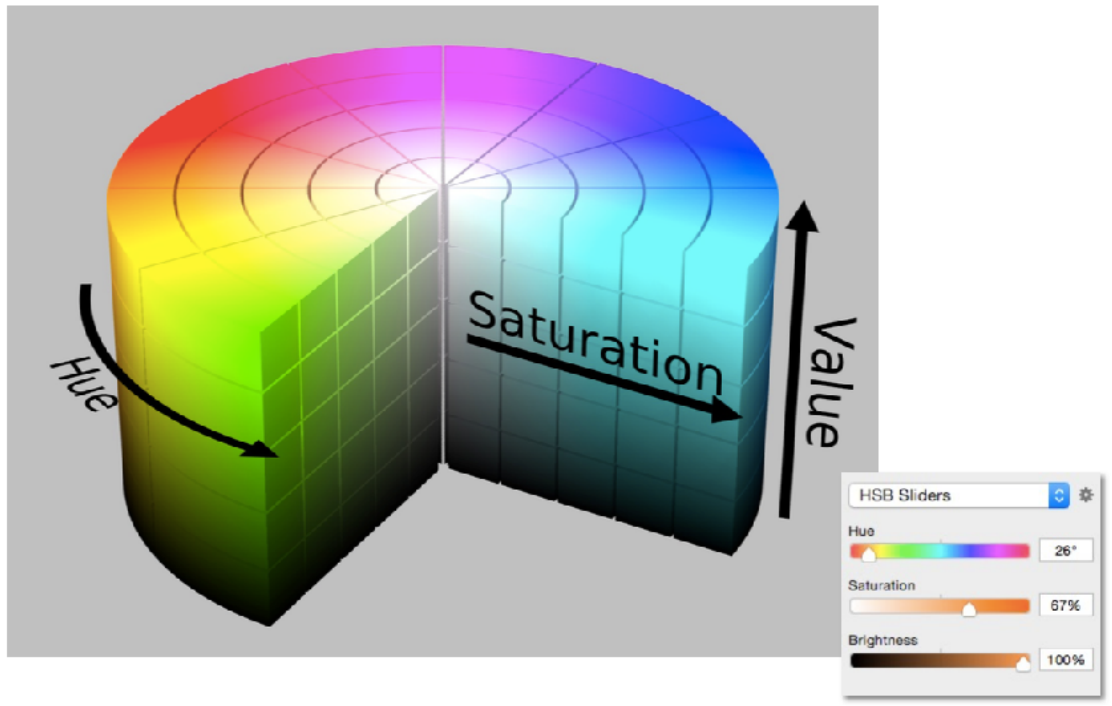
Axes of space correspond to natural notions of "characteristics" of color





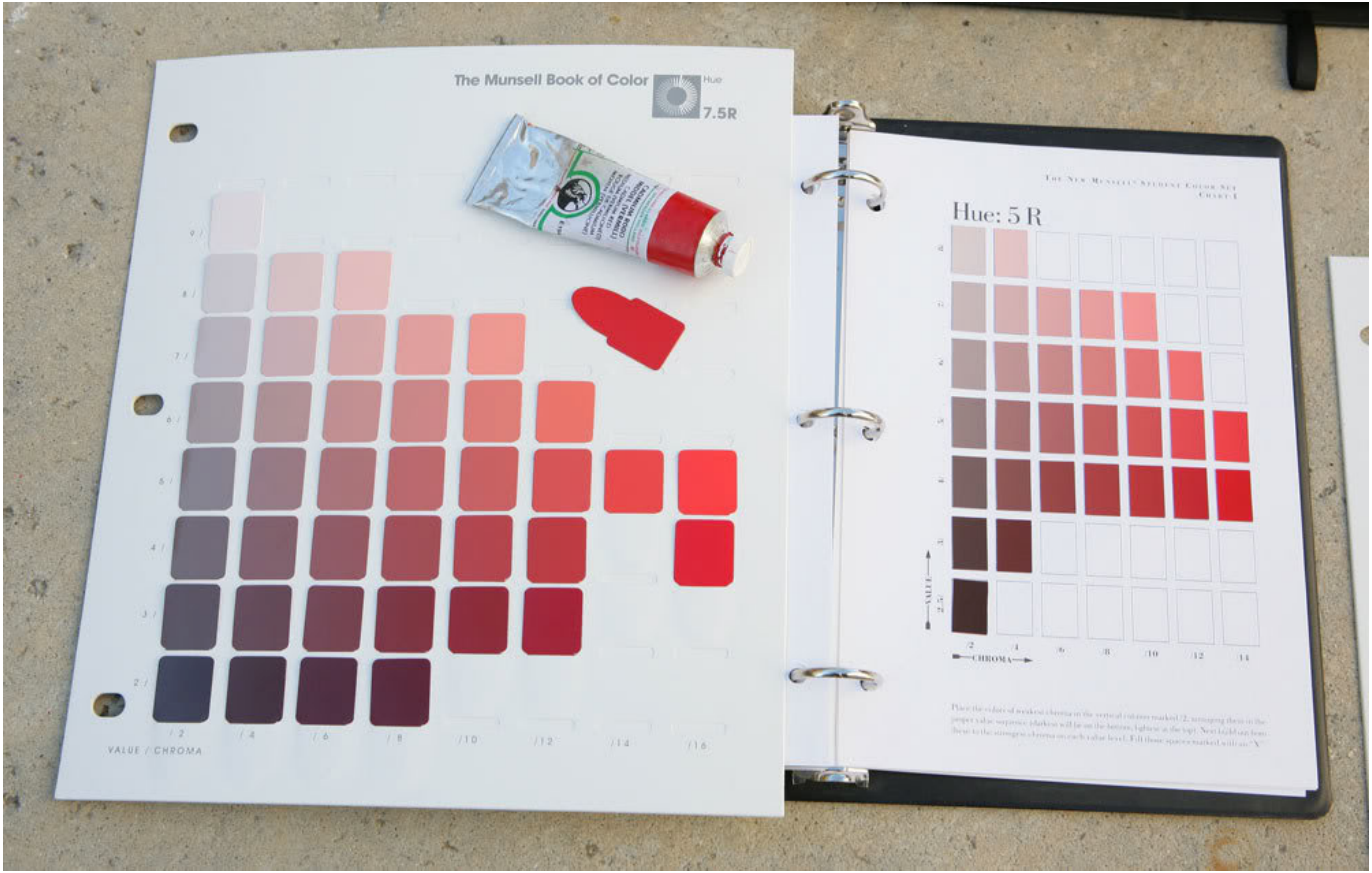
Perceptual dimensions of color

- **Hue**
  - the “kind” of color, regardless of attributes
  - colorimetric correlate: dominant wavelength
  - artist’s correlate: the chosen pigment color



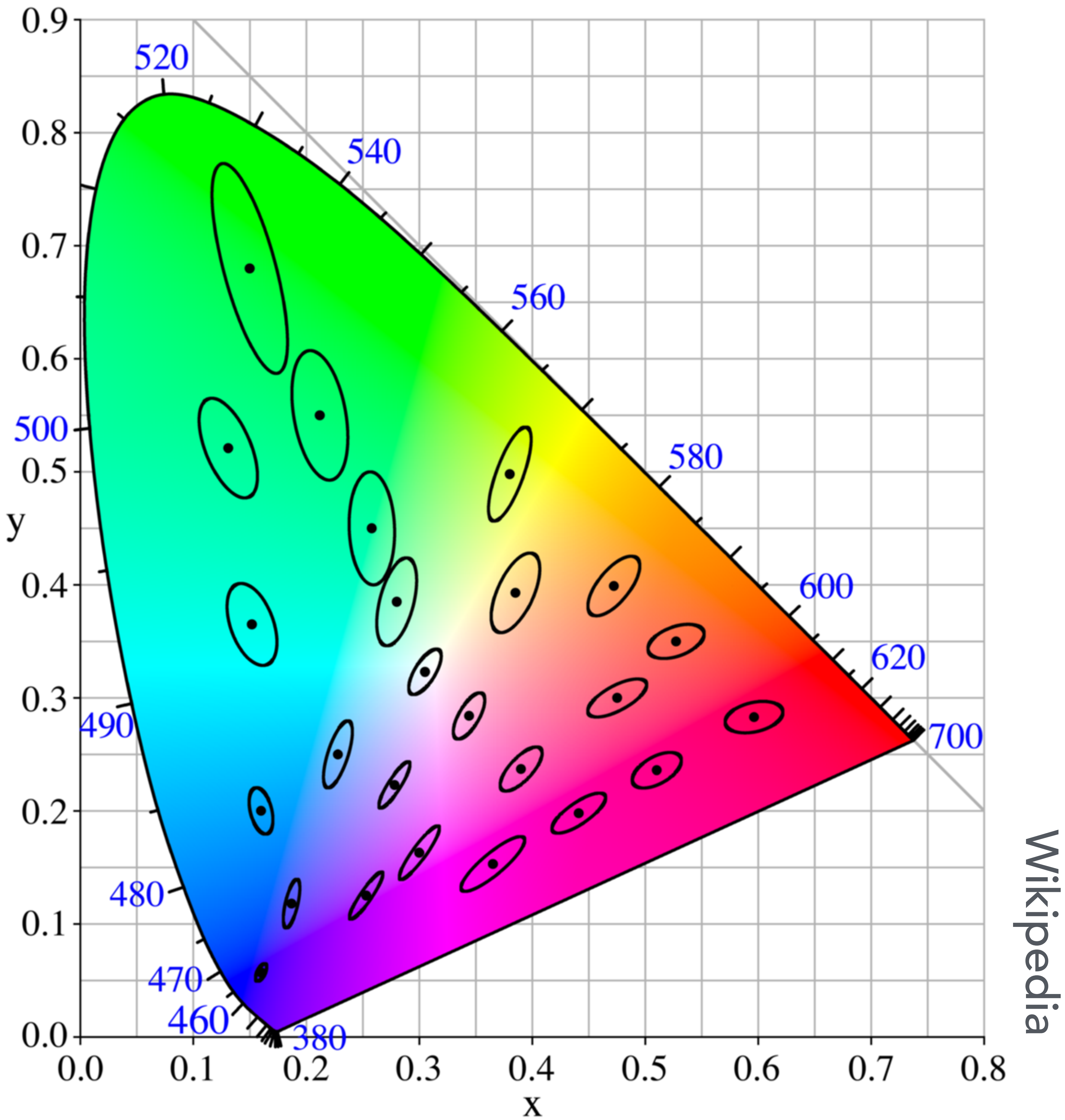
- **Saturation**
  - the “colorfulness”
  - colorimetric correlate: purity
  - artist’s correlate: fraction of paint from the colored tube
- **Lightness (or value)**
  - the overall amount of light
  - colorimetric correlate: luminance
  - artist’s correlate: tints are lighter, shades are darker

Munsell book of color



Swatch identified by three numbers: hue, value (lightness), and chroma (color purity)

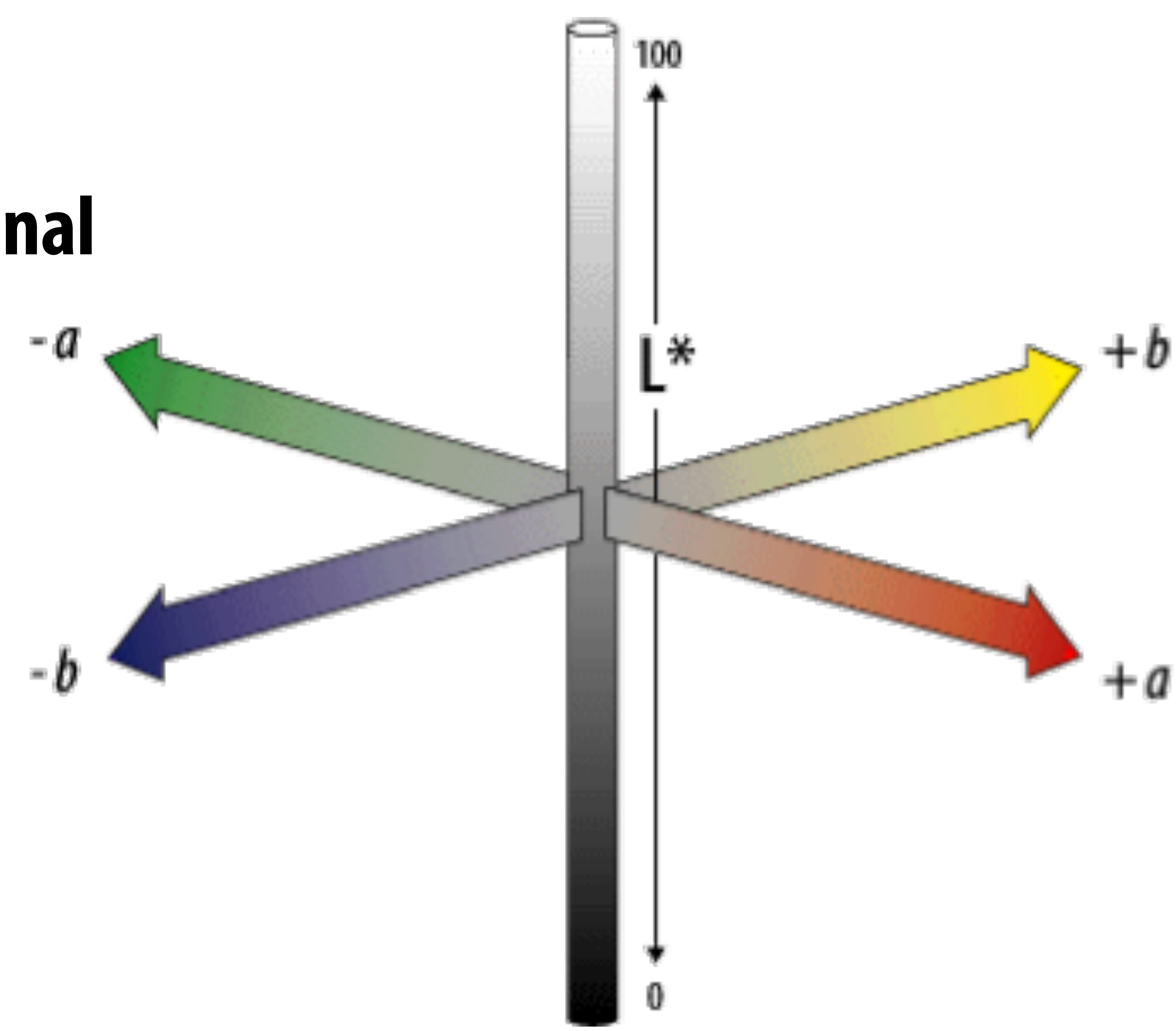
Perceptual non-uniformity



- In the xy chromaticity diagram at left, MacAdam ellipses show regions of perceptually equivalent color (ellipses enlarged 10x)
- Must non-linearly warp the diagram to achieve uniform perceptual distances

CIELAB Space (L*a*b*)

- A commonly used color space that strives for perceptual uniformity
  - L* is lightness
  - a* and b* are color-opponent pairs
    - a* is red-green, and b* is blue-yellow
  - A gamma transform is used for warping because perceived brightness is proportional to scene intensity$^{\gamma}$, where $\gamma \approx 1/3$

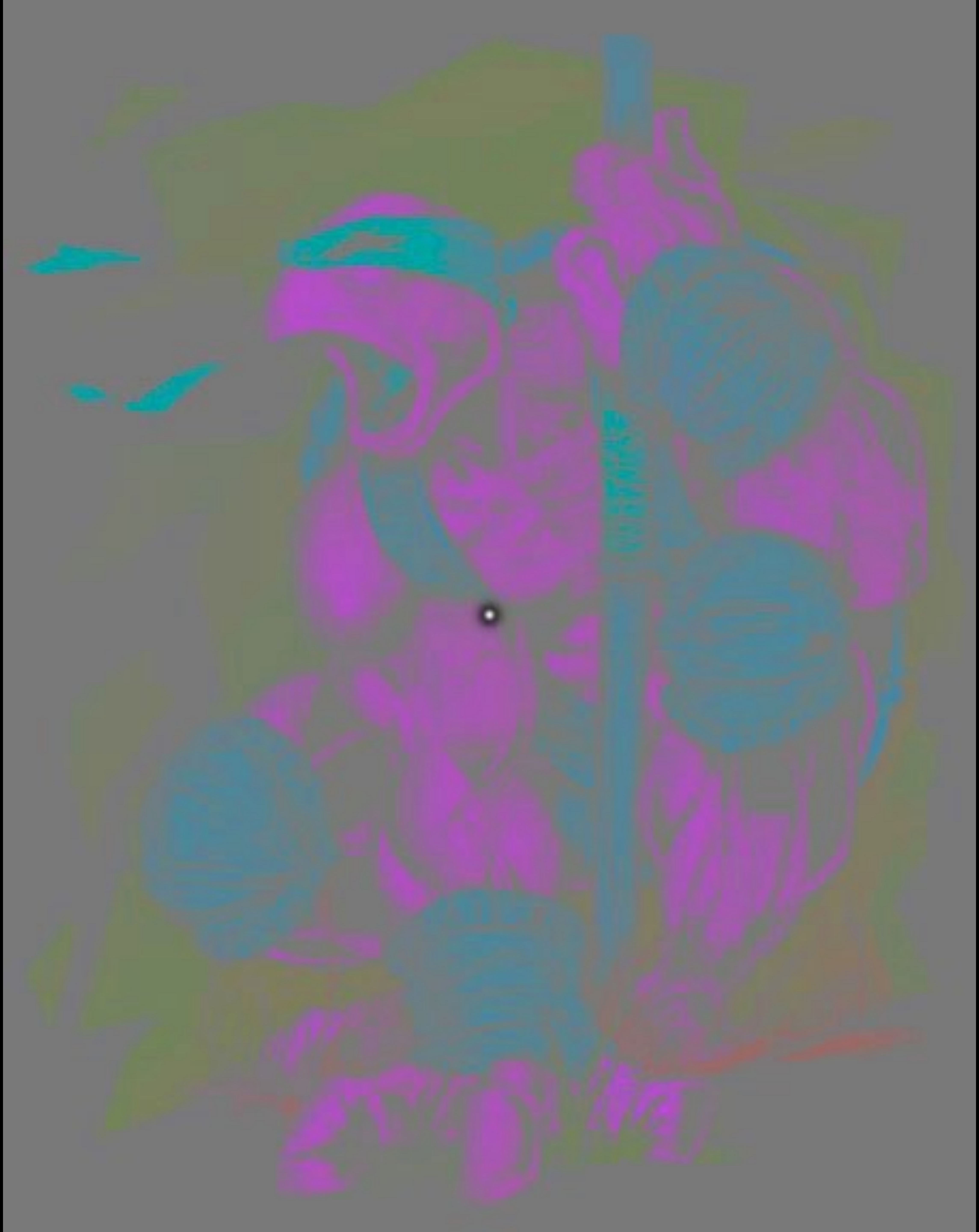


Opponent color theory

- There's a good neurological basis for the color space dimensions in CIE LAB
  - the brain seems to encode color early on using three axes:
    - white — black, red — green, yellow — blue
  - the white — black axis is lightness; the others determine hue and saturation
  - one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can't have a reddish green (just doesn't make sense)
    - thus red is the *opponent* to green
  - another piece of evidence: afterimages (following slides)

slide credit: Steve Marschner

Adapt



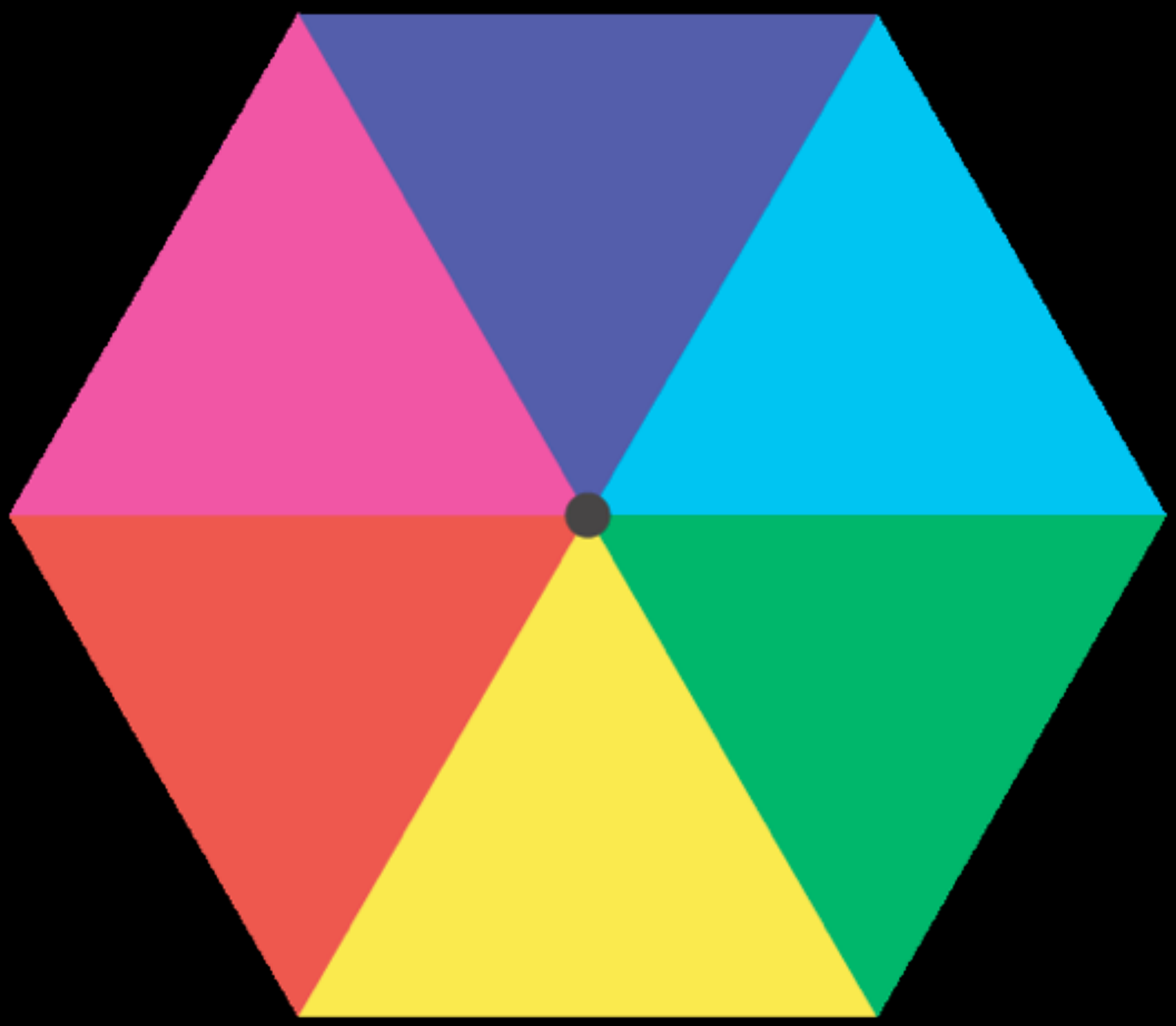
Adapt


keep staring at the black dot.

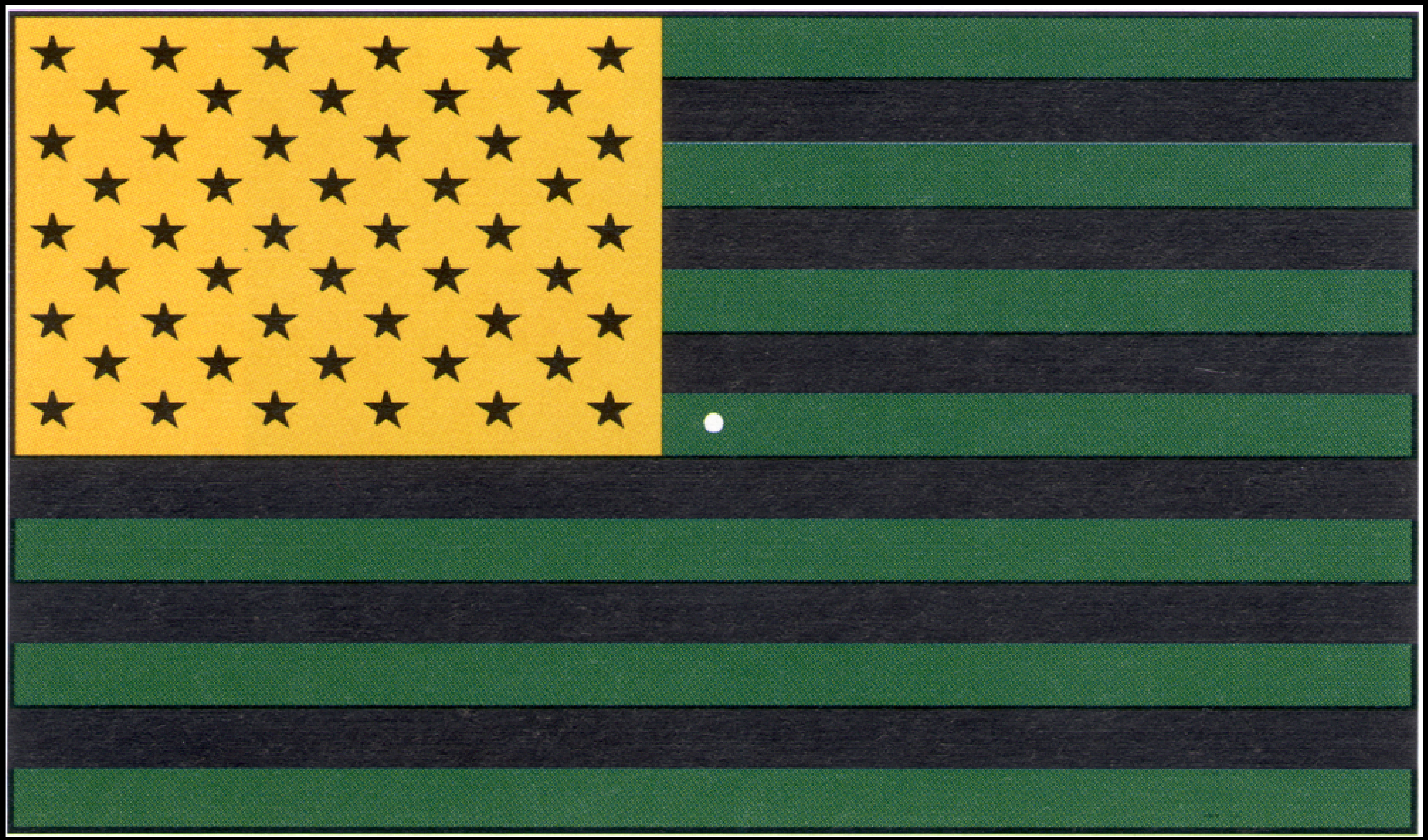

johnsadowski.com

[Slide left blank.. Students should see afterimage.]

Image

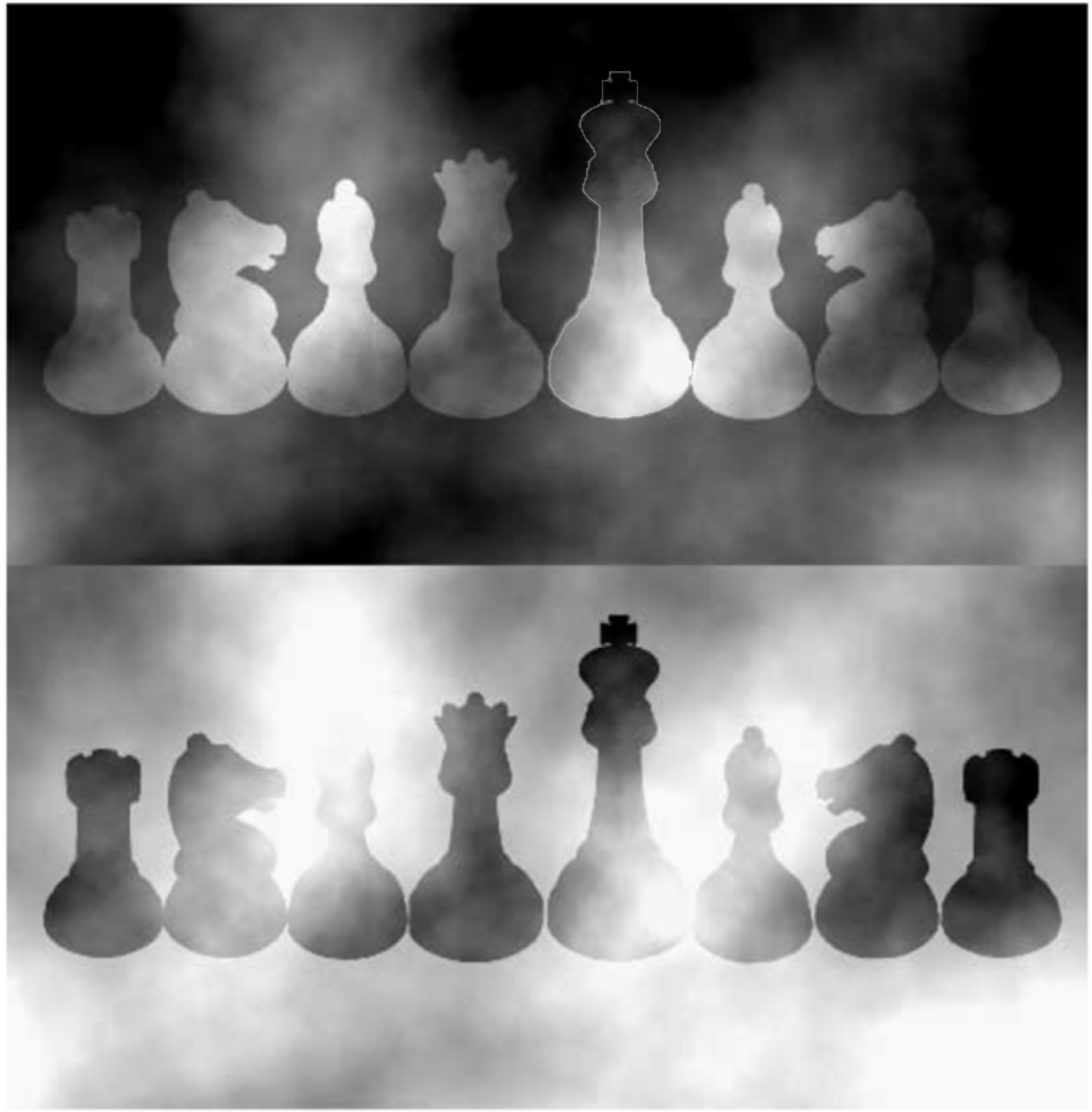


Afterimage

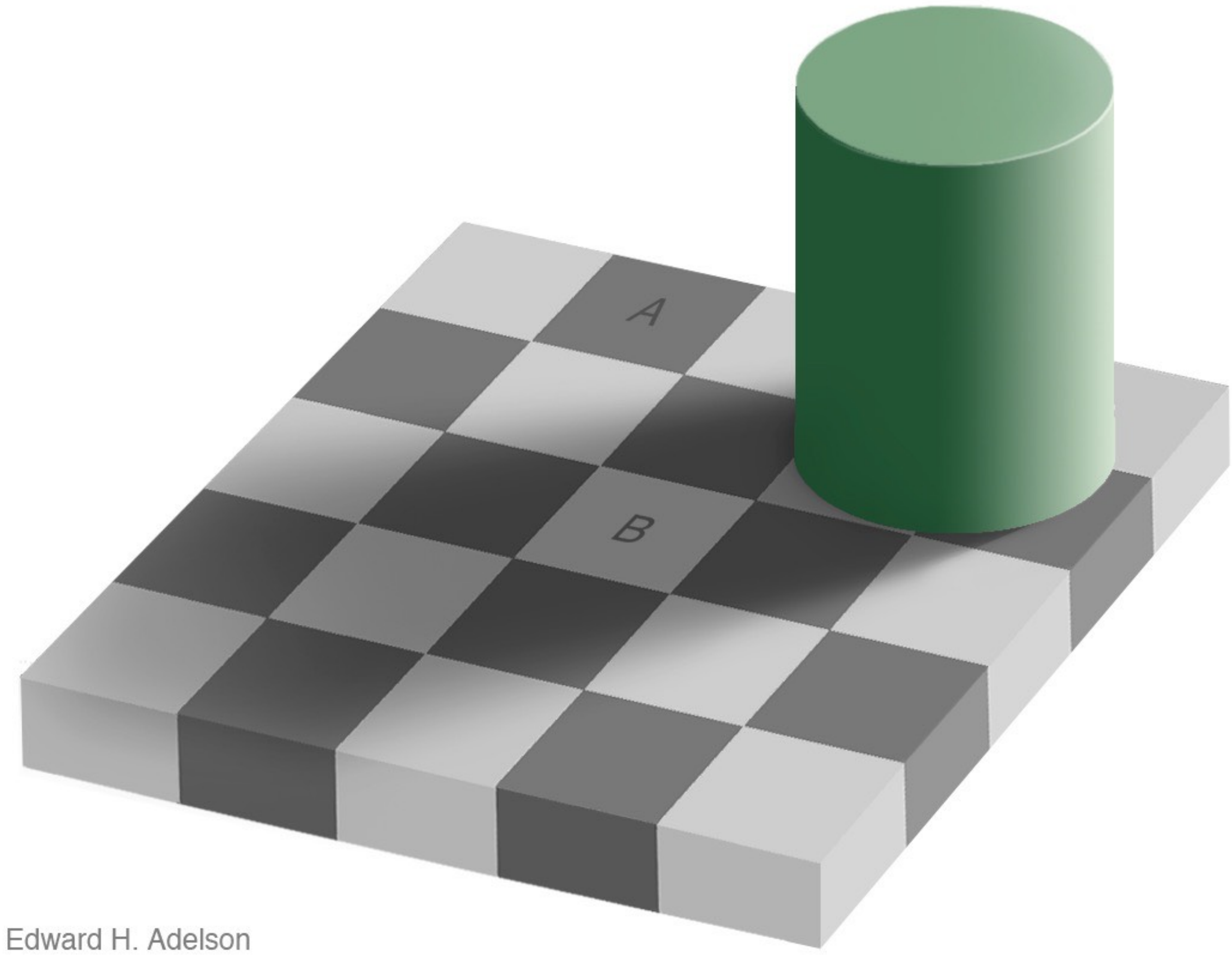


[Slide left blank.. Students should see afterimage.]

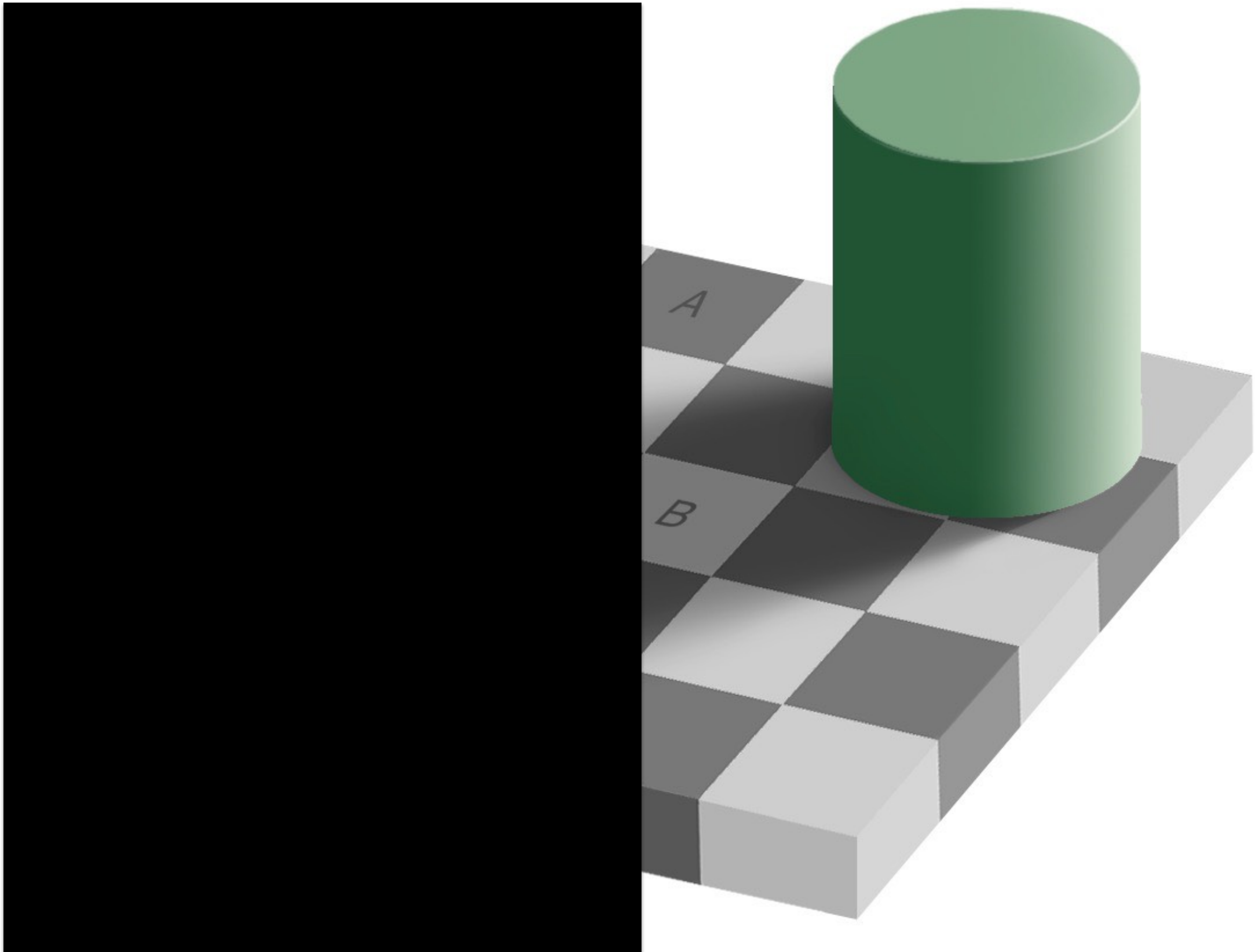
Even simple judgments – such as lightness - depend on brain processing (Anderson and Winawer, Nature, 2005)



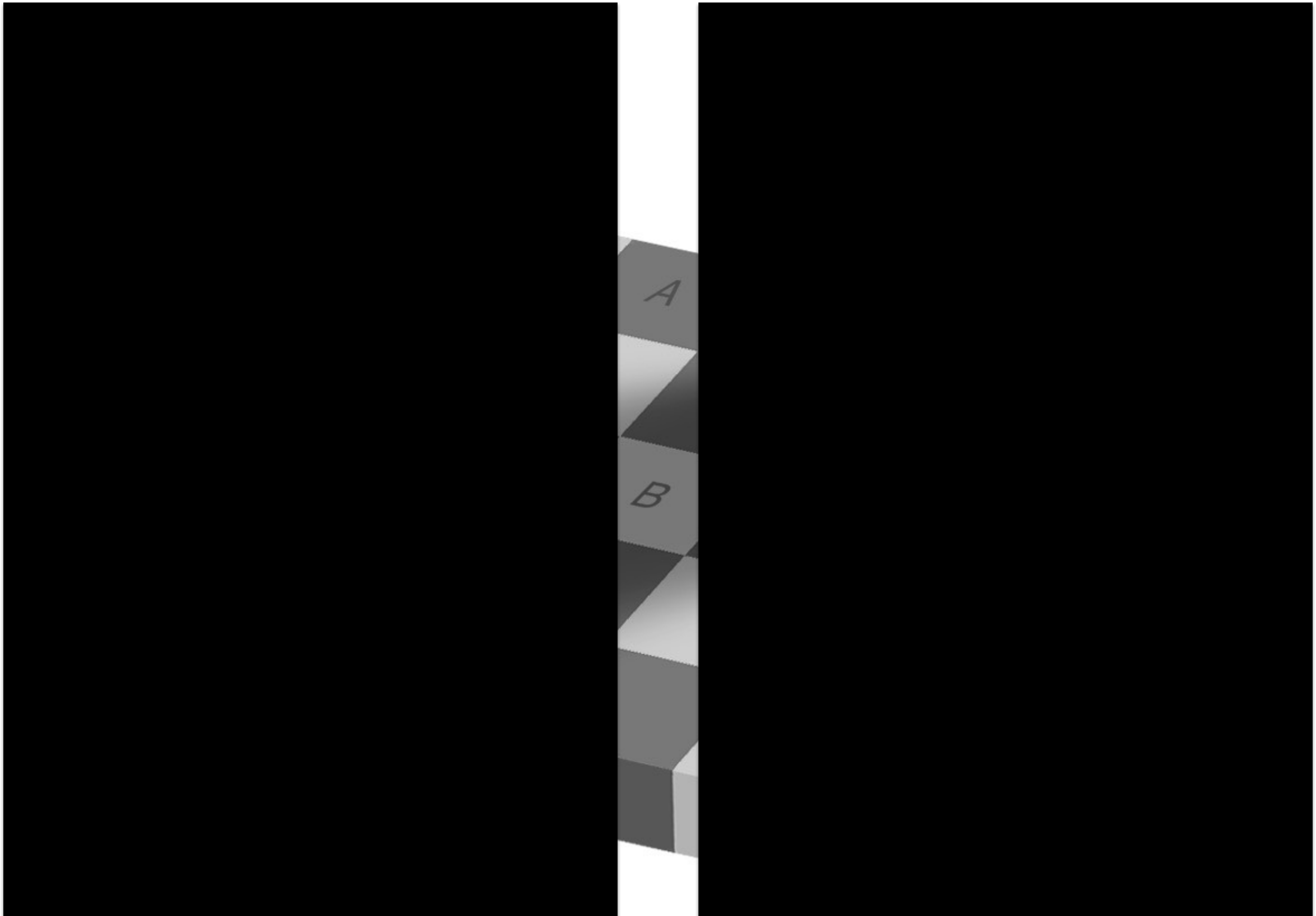
Everything is relative



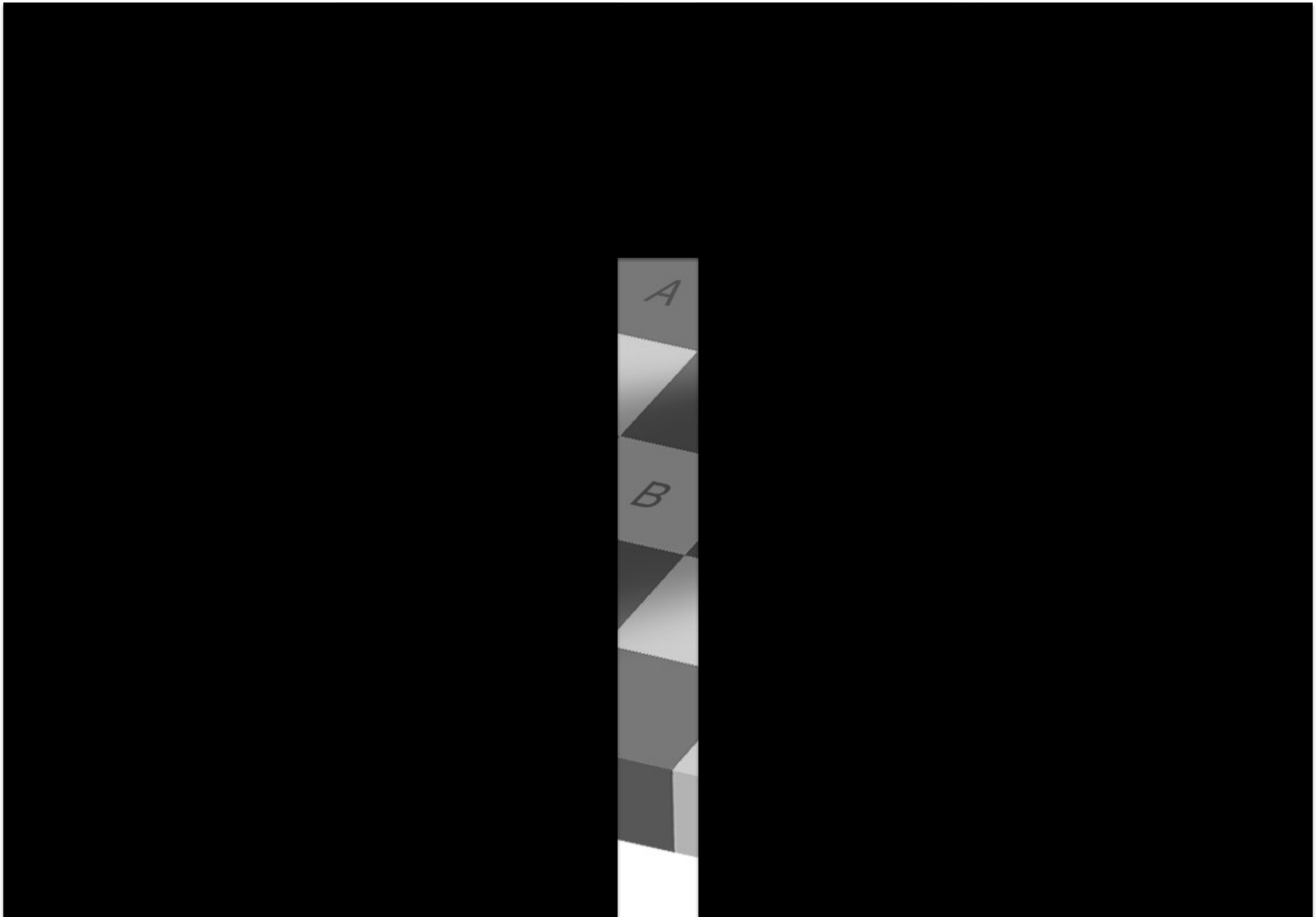
Everything is relative



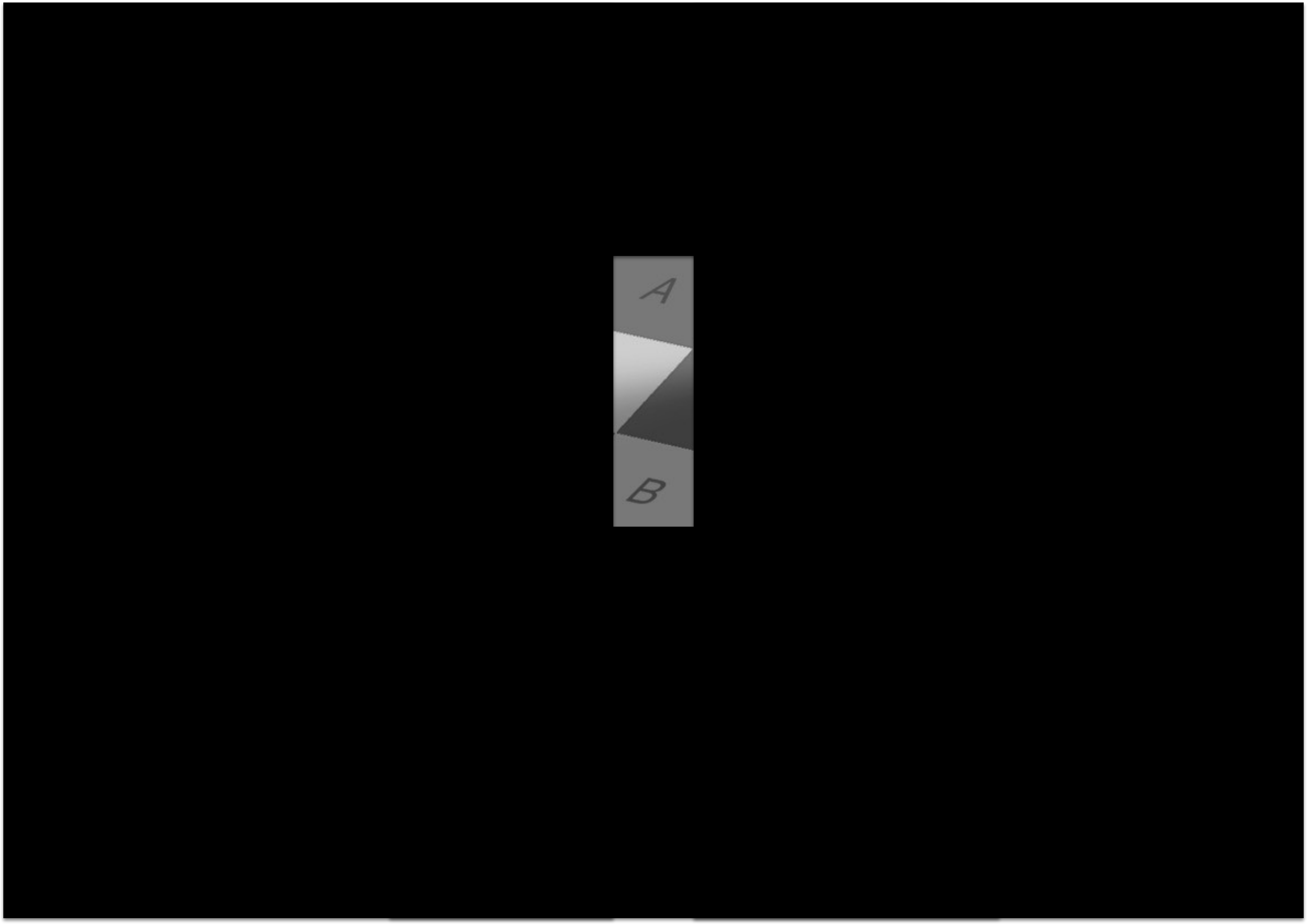
Everything is relative



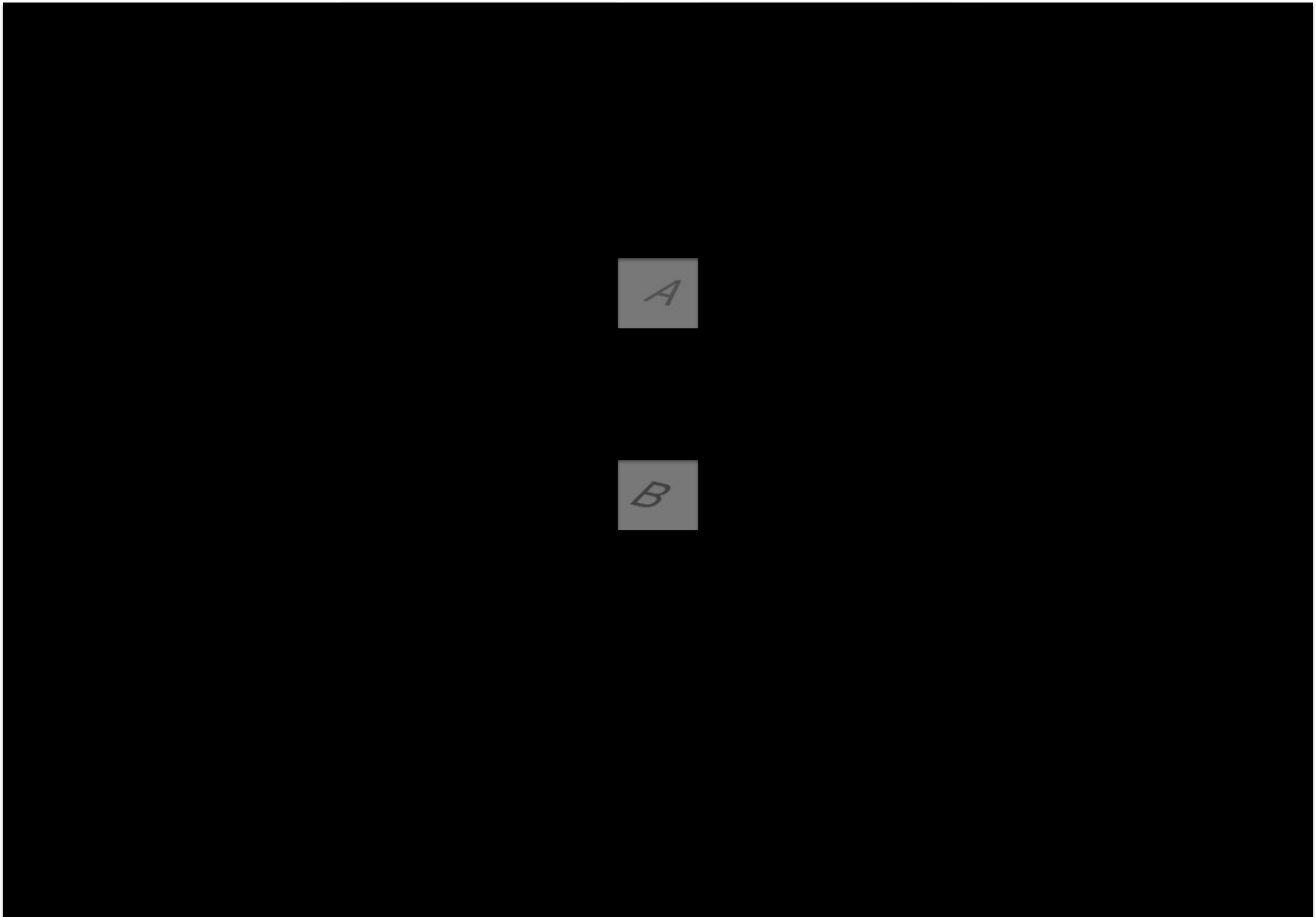
Everything is relative



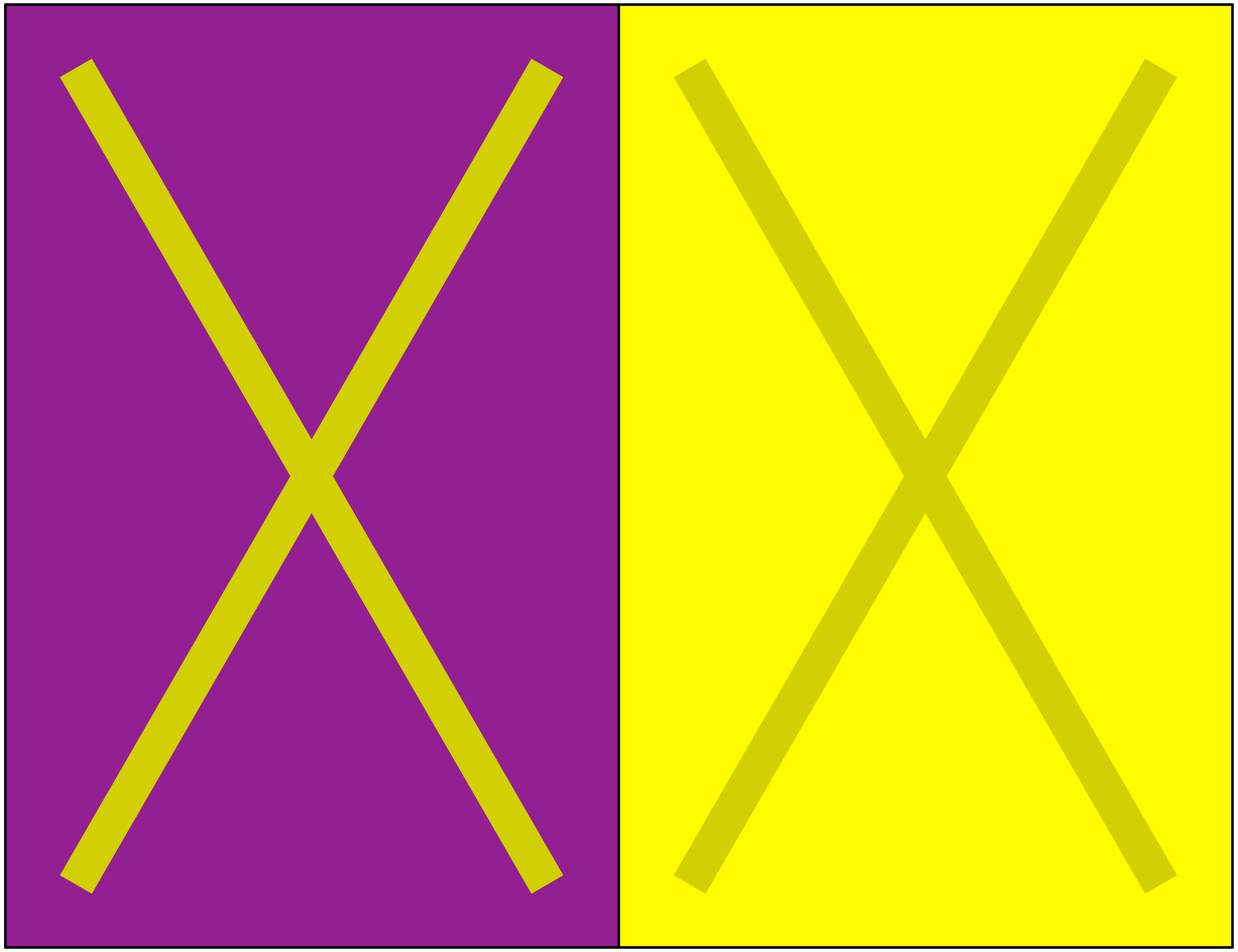
Everything is relative



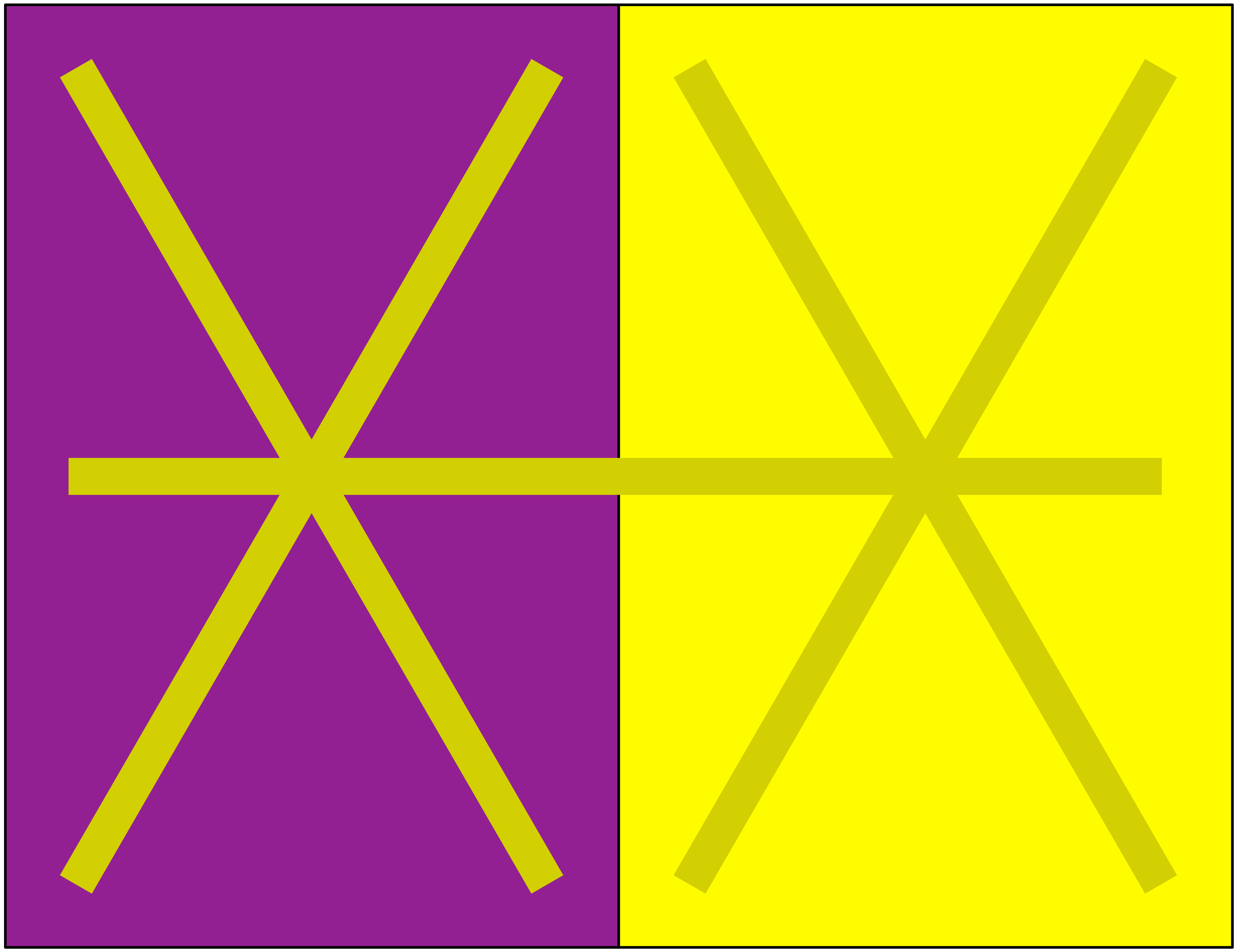
Everything is relative



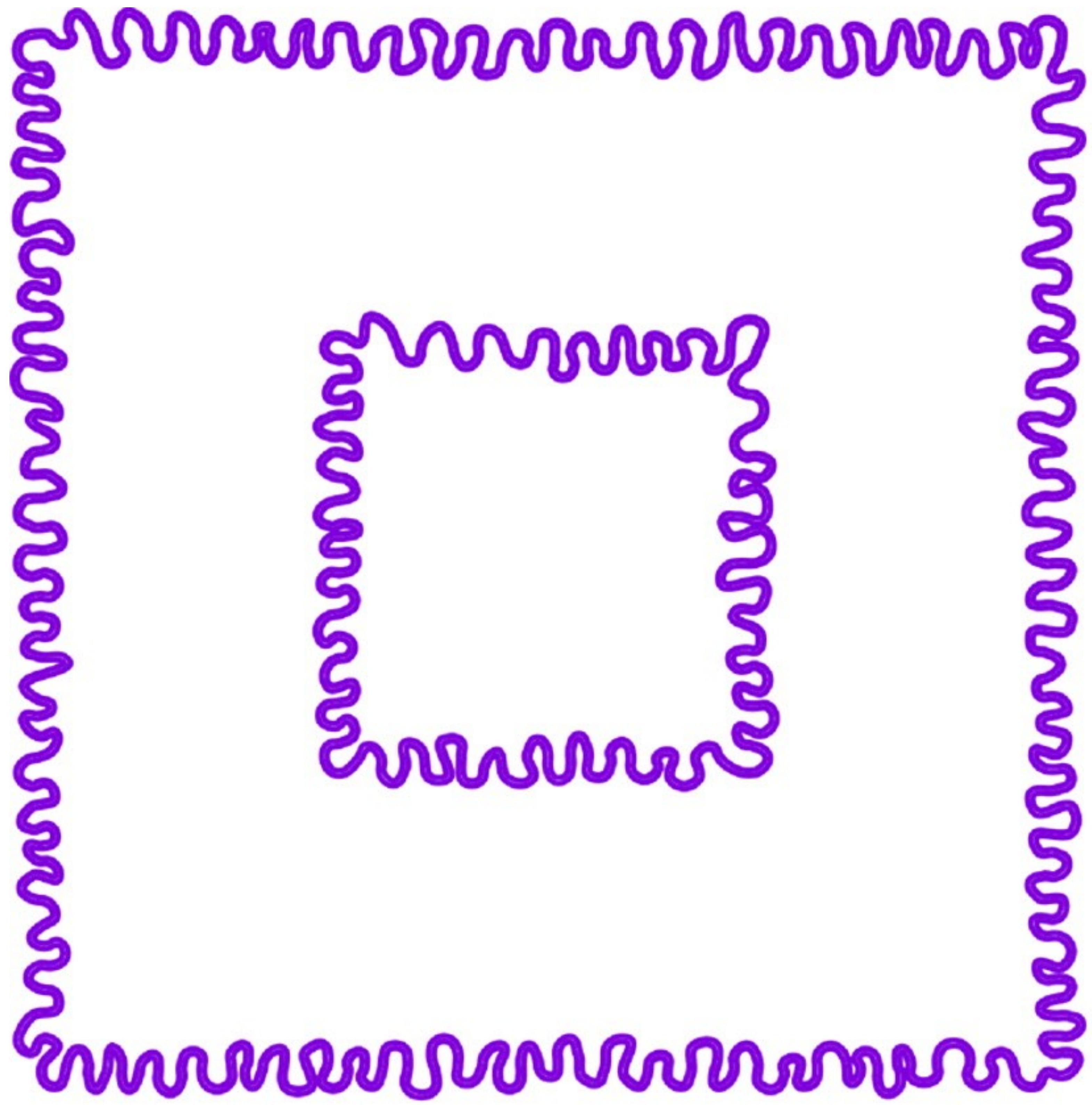
Everything is relative



Everything is relative

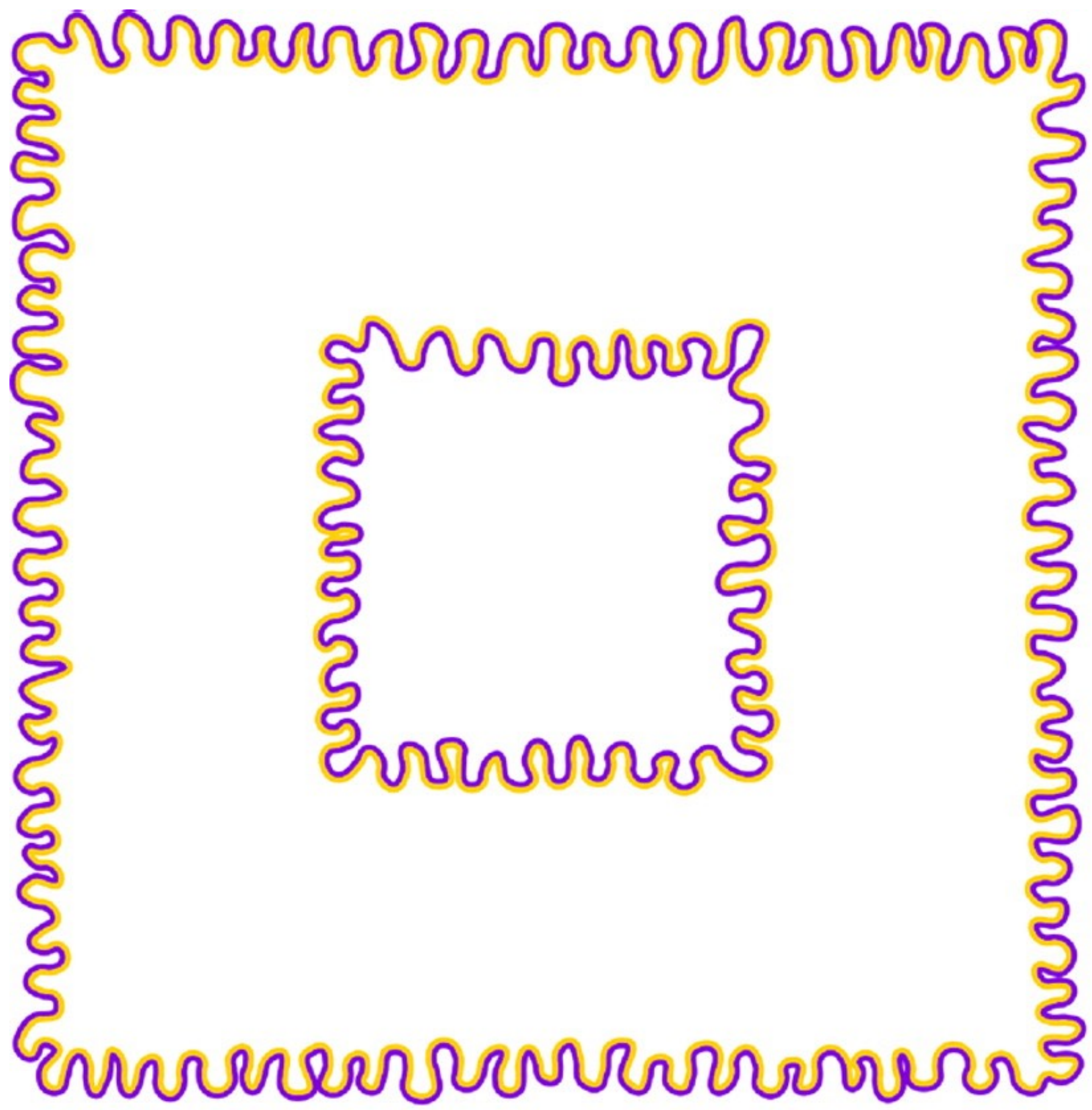


Watercolor illusion



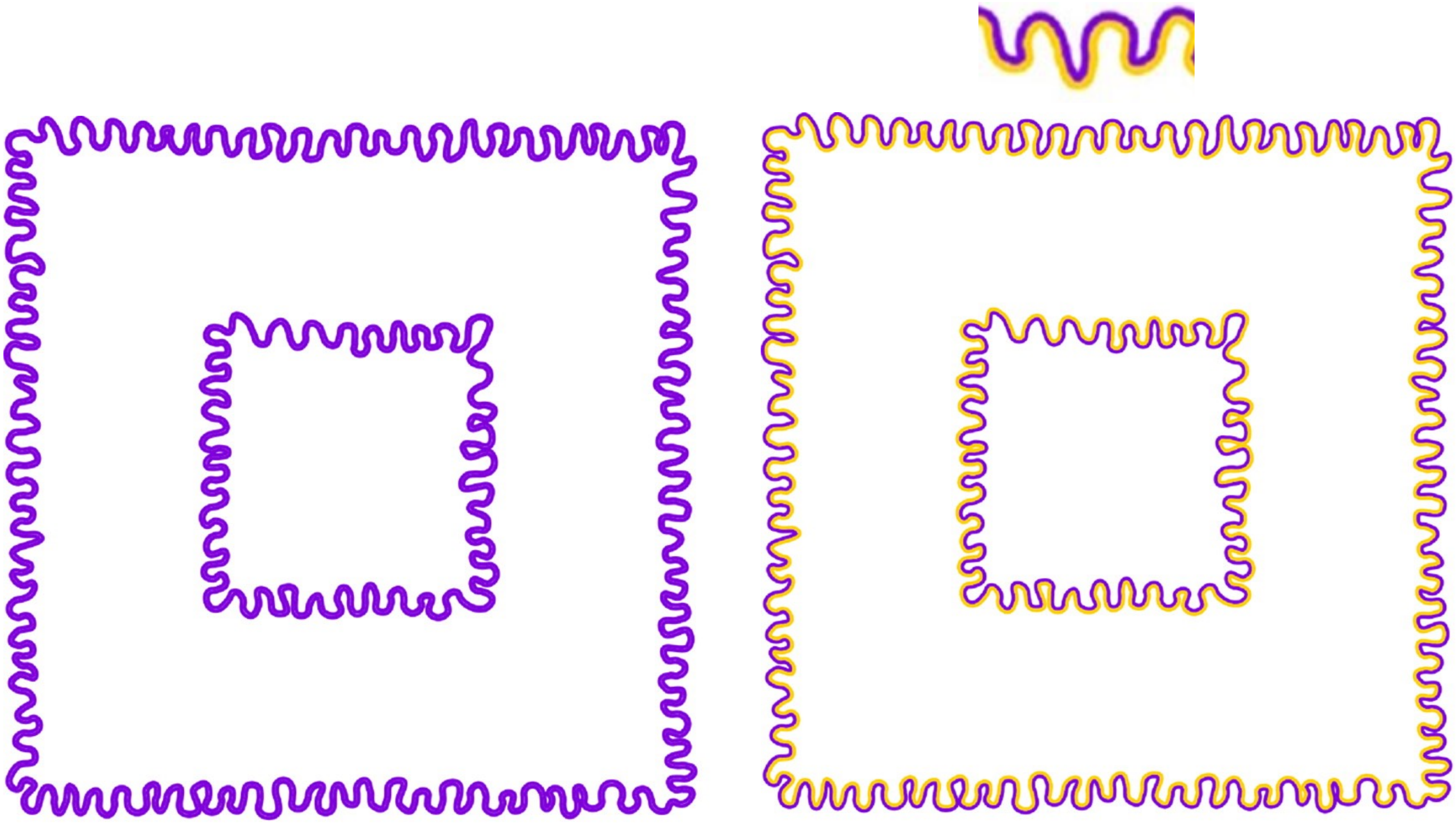
Pinna & Tanca, *JOV*, 2008

Watercolor illusion



Pinna & Tanca, *JOV*, 2008

Watercolor illusion



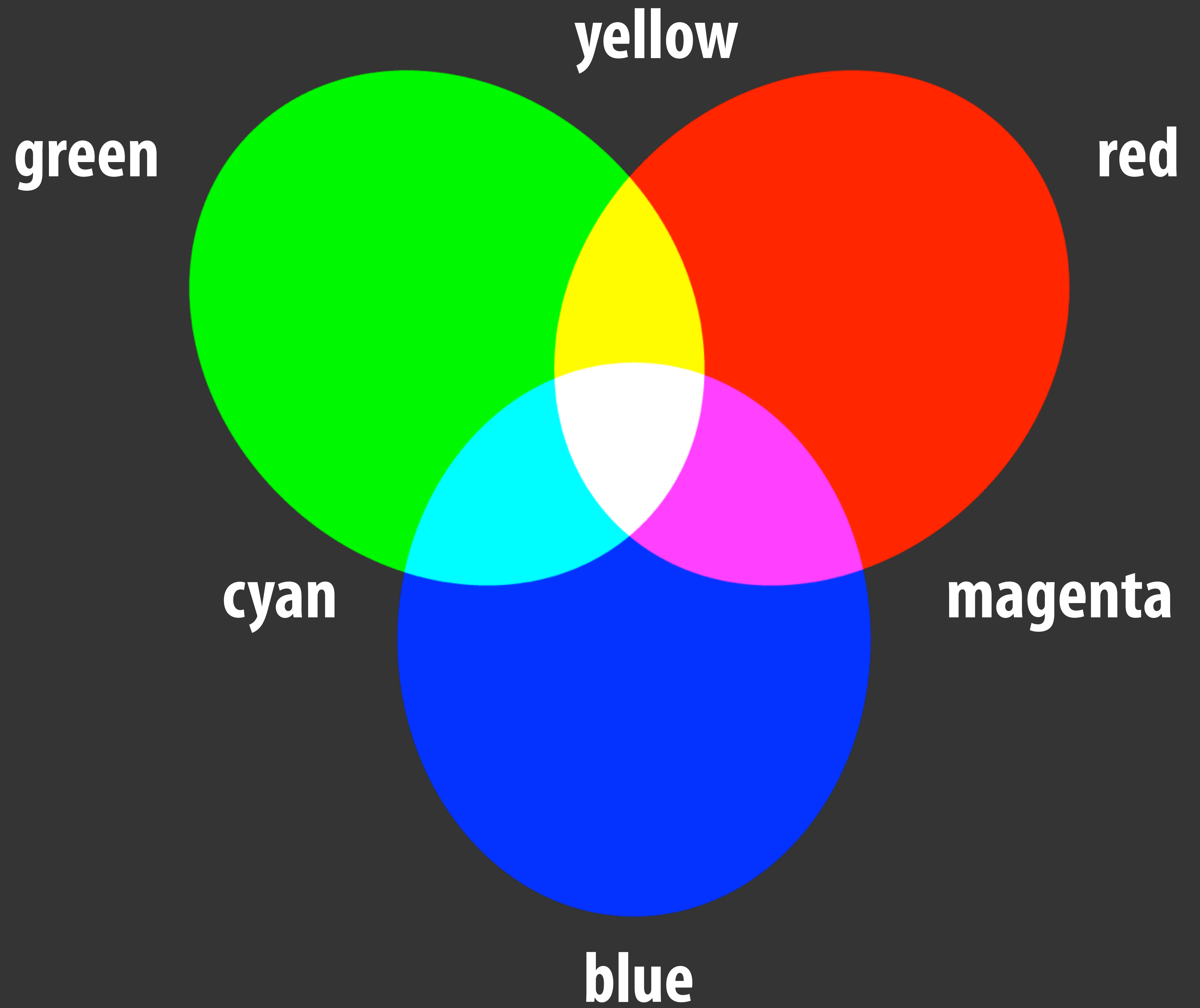
Pinna & Tanca, *JOV*, 2008

Things to remember

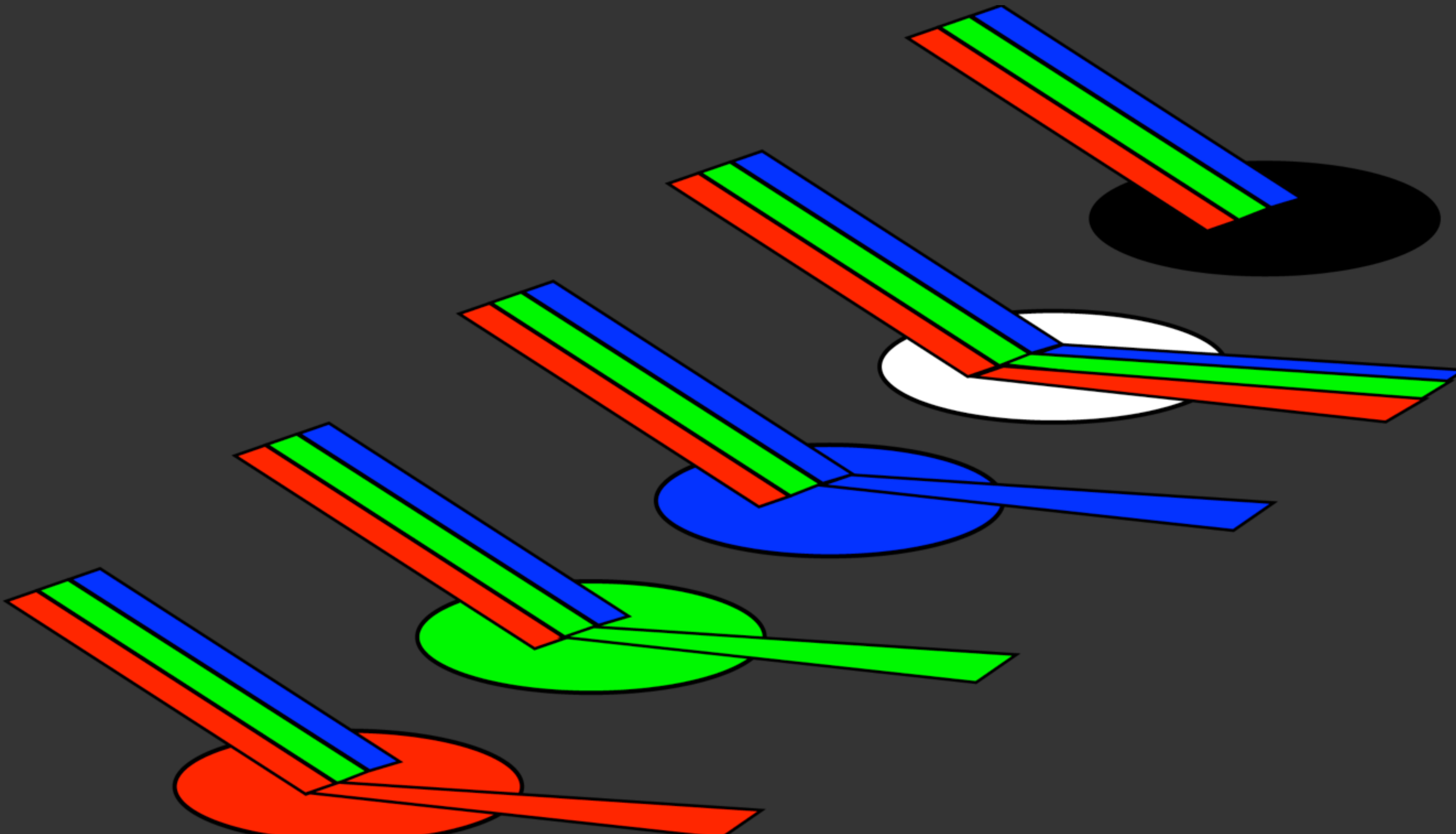
- Physics of Light
  - Spectral power distribution (SPD)
  - Superposition (linearity)
- Tristimulus theory of color
  - Spectral response of human cone cells (S, M, L)
  - Metamers - different SPDs with the same perceived color
  - Color reproduction mathematics
  - Color matching experiment, per-wavelength matching functions
- Color spaces
  - CIE RGB, XYZ, xy chromaticity, LAB, HSV
  - Gamut

If extra time:
subtractive color spaces

Adding light energy

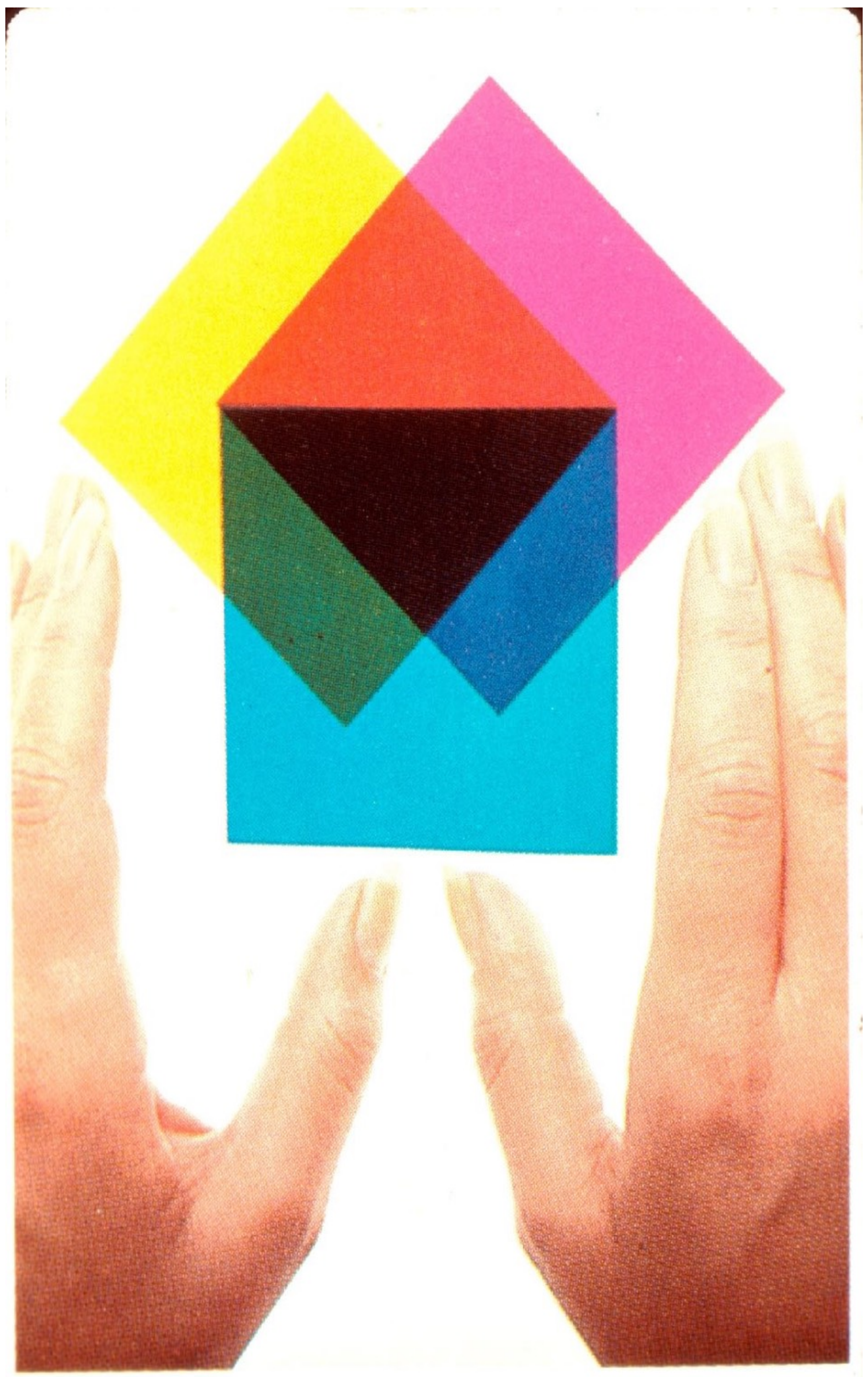


Subtracting light energy



shining white light on various colored pigments

Subtractive color



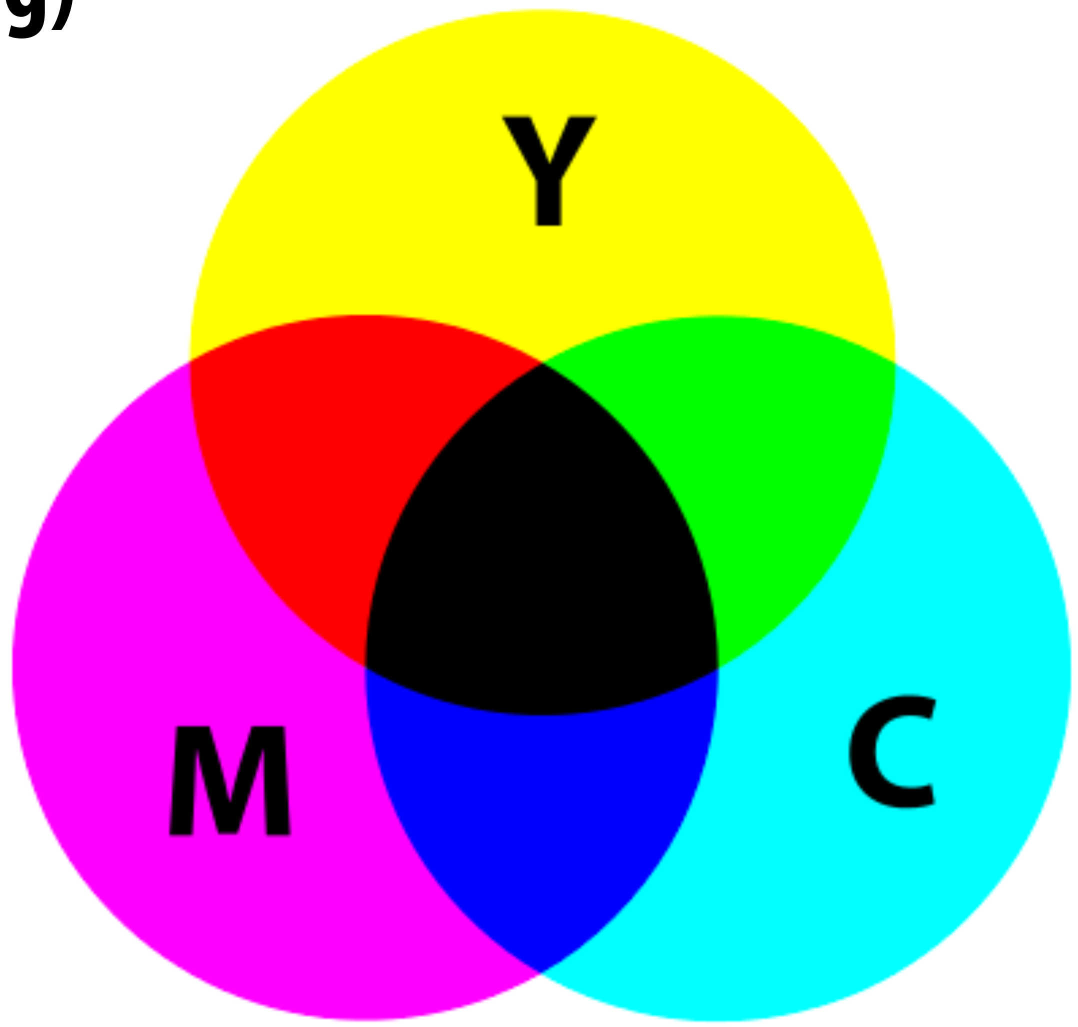
- Produce desired spectrum by subtracting from white light (usually via absorption by pigments)
- Photographic media (slides, prints) work this way
- Leads to C, M, Y (cyan, magenta, yellow) as primaries
- Approximately, 1 – R, 1 – G, 1 – B

[source unknown]

Subtractive color spaces

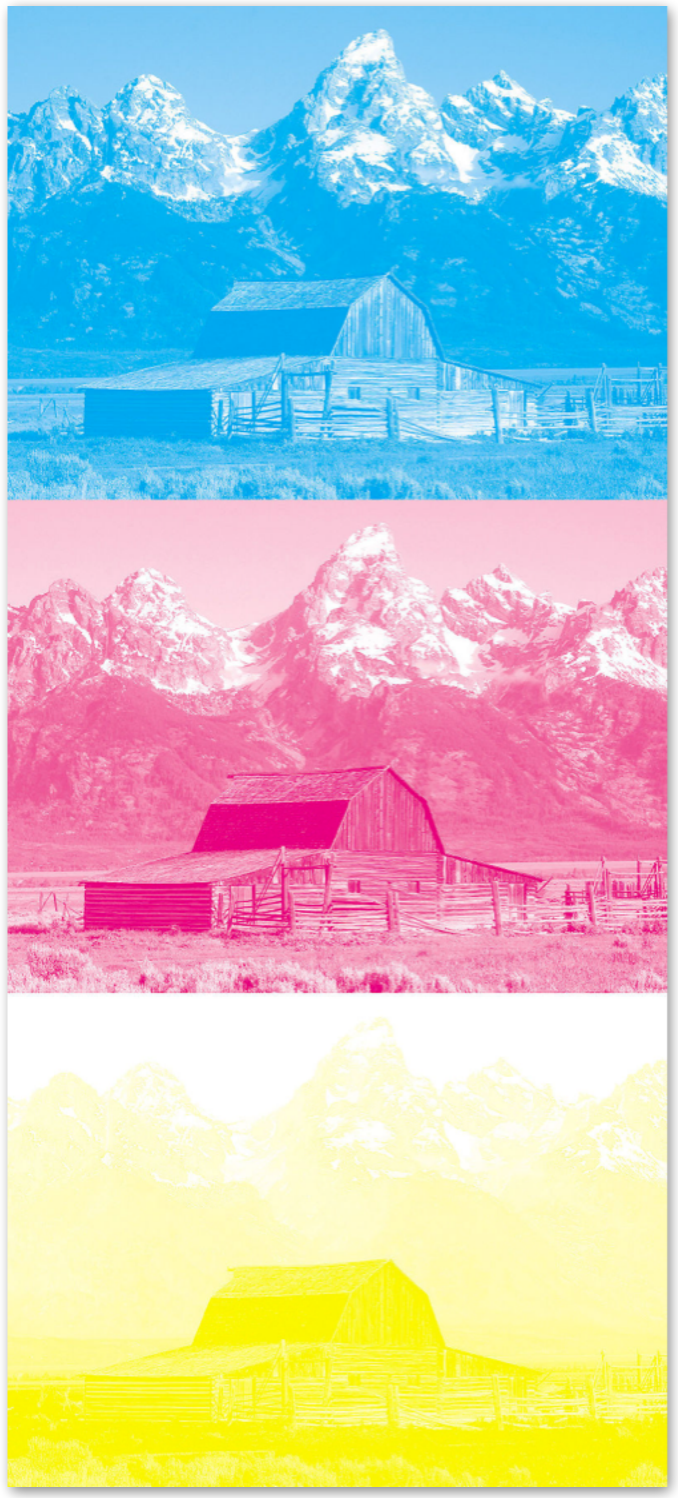
- Up to this point, we've described color in terms of superposition of primaries. (addition of primaries)
  - Good description of color formed by mixing light from multiple light sources (e.g., displays)

- When describing color of reflected light, each mixed primary contributes to absorption of light (requires a subtractive color space)
  - Describes how to form colors by mixing inks (e.g., printing)



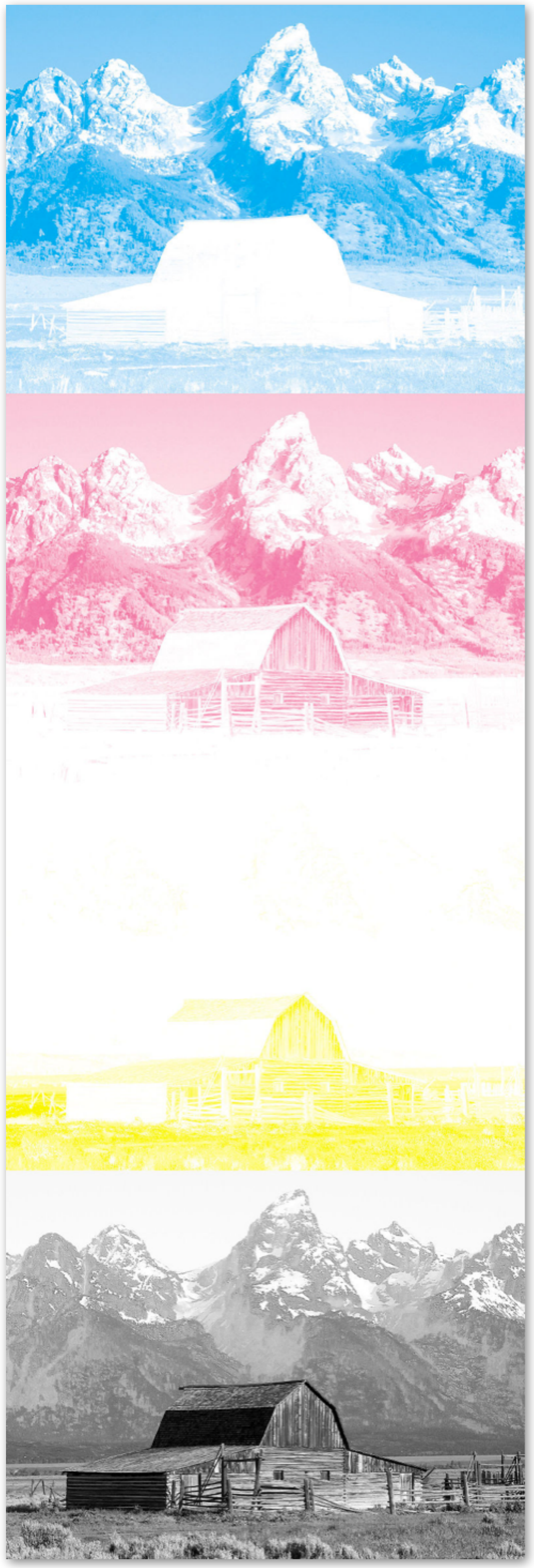
CMYK adds 4th component (K=black) as using a single black ink gives better black (and is more ink efficient) than mixing CMY primaries

CMY Representation



(0,0,0) is white
(1,1,1,) is black

CMYK Representation



Acknowledgements

- **Thanks and credit for slides to Ren Ng, Steve Marschner, Brian Wandell, Marc Levoy, Katherine Breeden, Austin Roorda, Keenan Crane, James O'Brien.**