Lecture 16:

Image Compression and Basic Image Processing

Interactive Computer Graphics
Stanford CS248, Winter 2019
Recurring themes in the course

- Choosing the right representation for a task
  - e.g., choosing the right basis

- Exploiting human perception for computational efficiency
  - Errors/approximations in algorithms can be tolerable if humans do not notice

- Convolution as a useful operator
  - To remove high frequency content from images
  - What else can we do with convolution?
Image Compression
A recent sunset in Half Moon Bay

Picture taken on my iPhone 7 (12 MPixel sensor)

4032 x 3024 pixels x (3 bytes/pixel) = 34.9 MB uncompressed image

JPG compressed image = 2.9 MB
Idea 1:

- What is the most efficient way to encode intensity values as a byte?
- Encode based on how the brain perceives brightness not, based on actual response of eye
Review from last time

- Sensor’s response is proportional to amount of light arriving at sensor

\[ R = \int_{\lambda} \Phi(\lambda) r(\lambda) d\lambda \]
**Lightness (perceived brightness) aka luma**

Lightness ($L^*$) \( \Leftarrow \) Lightness ($L^*$) (Perceived by brain)

Luminance ($Y$) \( \Rightarrow \) Luminance ($Y$) (Response of eye)

\[
L^* \propto Y^{0.4} \\
L^* \propto Y^{0.5}
\]

In a dark room, you turn on a light with luminance: \( Y_1 \)
You turn on a second light that is identical to the first. Total output is now: \( Y_2 = 2Y_1 \)

Total output appears \( 2^{0.4} = 1.319 \) times brighter to dark-adapted human

Note: Lightness ($L^*$) is often referred to as luma ($Y'$)
Consider an image with pixel values encoding luminance (linear in energy hitting sensor)

Consider 12-bit sensor pixel:
Can represent 4096 unique luminance values in output image

Values are ~ linear in luminance since they represent the sensor’s response
Problem: quantization error

Many common image formats store 8 bits per channel (256 unique values)
Insufficient precision to represent brightness in darker regions of image

Luminance (Y)

Perceived brightness: $L^*$

$L^* = Y^{0.45}$

Bright regions of image: perceived difference between pixels that differ by one step in luminance is small! (human may not even be able to perceive difference between pixels that differ by one step in luminance!)

Dark regions of image: perceived difference between pixels that differ by one step in luminance is large! (quantization error: gradients in luminance will not appear smooth.)

Rule of thumb: human eye cannot differentiate <1% differences in luminance
Store lightness, not luminance

Idea: distribute representable pixel values evenly with respect to perceived brightness, not evenly in luminance (make more efficient use of available bits)

Solution: pixel stores \( Y^{0.45} \)
Must compute \((\text{pixel\_value})^{2.2}\) prior to display on LCD

Warning: must take caution with subsequent pixel processing operations once pixels are encoded in a space that is not linear in luminance.

\text{e.g., When adding images should you add pixel values that are encoded as lightness or as luminance?}
Idea 2:

- Chrominance ("chroma") subsampling

- The human visual system is less sensitive to detail in chromaticity than in luminance
  - So it is sufficient to sample chroma at a lower rate
Recall from last time: RGB color space

Color defined by 3D point in space defined by red, green, and blue primaries.

red = (1,0,0)  green = (0,1,0)  blue = (0,0,1)

Image credit:
Recall from last time: same color is represented by different coordinates in other color spaces

Example: HSV (hue, saturation, value)
**Y’CbCr color space**

Y’ = luma: perceived luminance (non-linear)

Cb = blue-yellow deviation from gray

Cr = red-cyan deviation from gray

Conversion from R’G’B’ to Y’CbCr:

\[
Y' = 16 + \frac{65.738 \cdot R_D}{256} + \frac{129.057 \cdot G_D}{256} + \frac{25.064 \cdot B_D}{256}
\]

\[
C_B = 128 + \frac{-37.945 \cdot R_D}{256} + \frac{74.494 \cdot G_D}{256} + \frac{112.439 \cdot B_D}{256}
\]

\[
C_R = 128 + \frac{112.439 \cdot R_D}{256} - \frac{94.154 \cdot G_D}{256} + \frac{18.285 \cdot B_D}{256}
\]

Example: compression in Y’CbCr

Original picture of Kayvon
Example: compression in Y'CbCr

Contents of CbCr color channels downsampled by a factor of 20 in each dimension (400x reduction in number of samples)
Example: compression in Y'CbCr

Full resolution sampling of luma (Y')
Example: compression in Y’CbCr

Reconstructed result
(looks pretty good)
Chroma subsampling

Y’CbCr is an efficient representation for storage (and transmission) because Y’ can be stored at higher resolution than CbCr without significant loss in perceived visual quality.

<table>
<thead>
<tr>
<th>Y’00</th>
<th>Y’10</th>
<th>Y’20</th>
<th>Y’30</th>
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<tbody>
<tr>
<td>Cb00</td>
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<td>Cr00</td>
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<tr>
<td>Y’01</td>
<td>Y’11</td>
<td>Y’21</td>
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<tr>
<td>Cr01</td>
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4:2:2 representation:
Store Y’ at full resolution
Store Cb, Cr at full vertical resolution, but only half horizontal resolution

X:Y:Z notation:
X = width of block
Y = number of chroma samples in first row
Z = number of chroma samples in second row

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4:2:0 representation:
Store Y’ at full resolution
Store Cb, Cr at half resolution in both dimensions

Real-world 4:2:0 examples:
most JPG images and H.264 video
Blue-Ray
Idea 3:

- Low frequency content is predominant in the real world

- The human visual system is less sensitive to high frequency sources of error in images

- So a good compression scheme needs to accurately represent lower frequencies, but it can be acceptable to sacrifice accuracy in representing higher frequencies
Recall: frequency content of images

Spatial domain result

Spectrum
Recall: frequency content of images

Spatial domain result

Spectrum (after low-pass filter)
All frequencies above cutoff have 0 magnitude
Recall: frequency content of images

Spatial domain result (strongest edges)

Spectrum (after high-pass filter)
All frequencies below threshold have 0 magnitude
A recent sunset in Half Moon Bay
A recent sunset in Half Moon Bay  (with noise added)
A recent sunset in Half Moon Bay (with more noise added)
A recent sunset in Half Moon Bay
What is a good representation for manipulating frequency content of images?

Hint:
Image transform coding via discrete cosine transform (DCT)

8x8 pixel block
(64 coefficients of signal in “pixel basis”)

64 basis coefficients

64 cosine basis vectors
(each vector is 8x8 image)

\[
\text{basis}[i,j] = \cos \left( \frac{i}{N} \left( x + \frac{1}{2} \right) \right) \times \cos \left( \frac{j}{N} \left( y + \frac{1}{2} \right) \right)
\]

In practice: DCT applied to 8x8 pixel blocks of Y' channel, 16x16 pixel blocks of Cb, Cr (assuming 4:2:0)
Examples of other bases

This slide illustrates basis images for 4x4 block of pixels

Pixel Basis
(Compact: each coefficient in representation only effects a single pixel of output)

DCT
Walsh-Hadamard
Haar Wavelet

[Image credit: https://people.xiph.org/~xiphmont/demo/daala/demo3.shtml]
Quantization produces small values for coefficients (only few bits needed per coefficient)
Quantization zeros out many coefficients

Result of DCT
(representation of image in cosine basis)

Quantization Matrix

Changing JPEG quality setting in your favorite photo app modifies this matrix ("lower quality" = higher values for elements in quantization matrix)
JPEG compression artifacts

Noticeable 8x8 pixel block boundaries

Noticeable error near high gradients

Low Quality

Medium Quality

Low-frequency regions of image represented accurately even under high compression
Why might JPEG compression not be a good compression scheme for illustrations and rasterized text?
Images with high frequency content do not exhibit as high compression ratios. Why?

Original image: 2.9MB JPG

Medium noise: 22.6 MB JPG

High noise: 28.9 MB JPG

Photoshop JPG compression level = 10 used for all compressed images

Uncompressed image:
4032 x 3024 x 24 bytes/pixel = 36.6 MB
Lossless compression of quantized DCT values

```
-26 -3 -6 2 2 -1 0 0
 0 -2 -4 1 1 0 0 0
-3 1 5 -1 -1 0 0 0
-4 1 2 -1 0 0 0 0
 1 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0
```

Quantized DCT Values

Entropy encoding: (lossless)

- Reorder values
- Run-length encode (RLE) 0’s
- Huffman encode non-zero values

JPEG compression summary

Coefficient reordering

DCT

Quantization Matrix

= phone quantization loses information (lossy compression!)

Quantized DCT

RLLE compression of zeros

Entropy compression of non-zeros

Compressed bits

Lossless compression!
JPEG compression summary

Convert image to Y’CbCr

Downsample CbCr (to 4:2:2 or 4:2:0)  (information loss occurs here)

For each color channel (Y’, Cb, Cr):

   For each 8x8 block of values
       Compute DCT
       Quantize results  (information loss occurs here)
       Reorder values
       Run-length encode 0-spans
       Huffman encode non-zero values
Key idea: exploit characteristics of human perception to build efficient image storage and image processing systems

- Separation of luminance from chrominance in color representation (Y’CrCb) allows reduced resolution in chrominance channels (4:2:0)

- Encode pixel values linearly in lightness (perceived brightness), not in luminance (distribute representable values uniformly in perceptual space)

- JPEG compression significantly reduces file size at cost of quantization error in high spatial frequencies
  - Human brain is more tolerant of errors in high frequency image components than in low frequency ones
  - Images of the real world are dominated by low-frequency components
Image processing basics
Example image processing operations

Increase contrast
Increasing contrast with “S curve”

Per-pixel operation:
output(x,y) = f(input(x,y))
Example image processing operations

Image Invert:
\[ \text{out}(x,y) = 1 - \text{in}(x,y) \]
Example image processing operations

Blur
Example image processing operations

Sharpen
Edge detection
A “smarter” blur (doesn’t blur over edges)
Review: convolution

\[(f \ast g)(x) = \int_{-\infty}^{\infty} f(y) g(x - y) \, dy\]

output signal \hspace{1cm} filter \hspace{1cm} input signal

It may be helpful to consider the effect of convolution with the simple unit-area “box” function:

\[f(x) = \begin{cases} 1 & \text{if } |x| \leq 0.5 \\ 0 & \text{otherwise} \end{cases}\]

\[(f \ast g)(x) = \int_{-0.5}^{0.5} g(x - y) \, dy\]

\(f \ast g\) is a “blurred” version of \(g\)
**Discrete 2D convolution**

\[(f \ast g)(x, y) = \sum_{i,j=-\infty}^{\infty} f(i, j)I(x - i, y - j)\]

Consider \(f(i, j)\) that is nonzero only when: 
\[-1 \leq i, j \leq 1\]

Then:
\[(f \ast g)(x, y) = \sum_{i,j=-1}^{1} f(i, j)I(x - i, y - j)\]

And we can represent \(f(i,j)\) as a 3x3 matrix of values where:

\[f(i, j) = F_{i,j}\] 

(often called: “filter weights”, “filter kernel”)
Simple 3x3 box blur

float input[(WIDTH+2) * (HEIGHT+2)];
float output[WIDTH * HEIGHT];
float weights[] = {1./9, 1./9, 1./9,
                  1./9, 1./9, 1./9,
                  1./9, 1./9, 1./9};

for (int j=0; j<HEIGHT; j++) {
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            for (int ii=0; ii<3; ii++)
                tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
        output[j*WIDTH + i] = tmp;
    }
}
7x7 box blur

Original

Blurred
Gaussian blur

- Obtain filter coefficients by sampling 2D Gaussian function

\[ f(i, j) = \frac{1}{2\pi\sigma^2} e^{-\frac{i^2 + j^2}{2\sigma^2}} \]

- Produces weighted sum of neighboring pixels (contribution falls off with distance)
  - In practice: truncate filter beyond certain distance for efficiency

\[
\begin{bmatrix}
.075 & .124 & .075 \\
.124 & .204 & .124 \\
.075 & .124 & .075
\end{bmatrix}
\]
7x7 gaussian blur

Original

Blurred
What does convolution with this filter do?

\[
\begin{bmatrix}
0 & -1 & 0 \\
-1 & 5 & -1 \\
0 & -1 & 0
\end{bmatrix}
\]

Sharpens image!
3x3 sharpen filter

Original

Sharpened
Another way to think about sharpening

- Let $I$ be the original image
- High frequencies in image $I = I - \text{blur}(I)$
- Sharpened image $= I + (I-\text{blur}(I))$
Original image (I)

Image credit: Kayvon’s parents
Blur(I)
I - blur(I)
$1 + (1 - \text{blur}(I))$
What does convolution with these filters do?

\[
\begin{bmatrix}
-1 & 0 & 1 \\
-2 & 0 & 2 \\
-1 & 0 & 1 \\
\end{bmatrix}
\]

Extracts horizontal gradients

\[
\begin{bmatrix}
-1 & -2 & -1 \\
0 & 0 & 0 \\
1 & 2 & 1 \\
\end{bmatrix}
\]

Extracts vertical gradients
Gradient detection filters

Note: you can think of a filter as a "detector" of a pattern, and the magnitude of a pixel in the output image as the "response" of the filter to the region surrounding each pixel in the input image (this is a common interpretation in computer vision).
Sobel edge detection

- Compute gradient response images

\[ G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \ast I \]

\[ G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \ast I \]

- Find pixels with large gradients

\[ G = \sqrt{G_x^2 + G_y^2} \]
Cost of convolution with N x N filter?

```c
float input[(WIDTH+2) * (HEIGHT+2)];
float output[WIDTH * HEIGHT];

float weights[] = {1./9, 1./9, 1./9,
                   1./9, 1./9, 1./9,
                   1./9, 1./9, 1./9};

for (int j=0; j<HEIGHT; j++) {
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            for (int ii=0; ii<3; ii++)
                tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
        output[j*WIDTH + i] = tmp;
    }
}
```

In this 3x3 box blur example:
Total work per image = 9 x WIDTH x HEIGHT

For N x N filter: N² x WIDTH x HEIGHT
Separable filter

A filter is separable if can be written as the outer product of two other filters. Example: a 2D box blur

\[
\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \ast \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}
\]

- Exercise: write 2D gaussian and vertical/horizontal gradient detection filters as product of 1D filters (they are separable!)

Key property: 2D convolution with separable filter can be written as two 1D convolutions!
Implementation of 2D box blur via two 1D convolutions

```c
int WIDTH = 1024
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float tmp_buf[WIDTH * (HEIGHT+2)];
float output[WIDTH * HEIGHT];
float weights[] = {1./3, 1./3, 1./3};

for (int j=0; j<(HEIGHT+2); j++)
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int ii=0; ii<3; ii++)
            tmp += input[j*(WIDTH+2) + i+ii] * weights[ii];
        tmp_buf[j*WIDTH + i] = tmp;
    }

for (int j=0; j<HEIGHT; j++) {
    for (int i=0; i<WIDTH; i++) {
        float tmp = 0.f;
        for (int jj=0; jj<3; jj++)
            tmp += tmp_buf[(j+jj)*WIDTH + i] * weights[jj];
        output[j*WIDTH + i] = tmp;
    }
}
```

Total work per image for N×N filter: 2N × WIDTH × HEIGHT

Stanford CS248, Winter 2019
Bilateral filter

Example use of bilateral filter: removing noise while preserving image edges

Bilateral filter

Example use of bilateral filter: removing noise while preserving image edges

http://opencvpython.blogspot.com/2012/06/smoothing-techniques-in-opencv.html
The bilateral filter is an “edge preserving” filter: down-weight contribution of pixels on the “other side” of strong edges. \( f(x) \) defines what “strong edge means”

- Spatial distance weight term \( f(x) \) could itself be a gaussian
  - Or very simple: \( f(x) = 0 \) if \( x > \text{threshold} \), \( 1 \) otherwise

Value of output pixel \((x,y)\) is the weighted sum of all pixels in the support region of a truncated gaussian kernel

But weight is combination of spatial distance and input image pixel intensity difference. (non-linear filter: like the median filter, the filter’s weights depend on input image content)
Bilateral filter

Input pixel $p$

Pixels with significantly different intensity as $p$ contribute little to filtered result (they are “on the “other side of the edge”

Input image
$G()$: gaussian about input pixel $p$
$f()$: Influence of support region

$G \times f$: filter weights for pixel $p$

Filtered output image

Figure credit: Durand and Dorsey, “Fast Bilateral Filtering for the Display of High-Dynamic-Range Images”, SIGGRAPH 2002
Bilateral filter: kernel depends on image content

Figure credit: SIGGRAPH 2008 Course: “A Gentle Introduction to Bilateral Filtering and its Applications” Paris et al.
Summary

- Last two lectures: representing images
  - Choice of color space (different representations of color)
  - Store values in perceptual space (non-linear in energy)
  - JPG image compression (tolerate loss due to approximate representation of high frequency components)

- Basic image processing operations
  - Per-pixel operations out(x,y) = f(in(x,y)) (e.g., contrast enhancement)
  - Image filtering via convolution (e.g., blur, sharpen, simple edge-detection)
  - Non-linear, data-dependent filters (median filter, avoid blurring over strong edges, etc.)