Lecture 5:
The Rasterization Pipeline
(and its implementation on GPUs)

Interactive Computer Graphics
Stanford CS248, Winter 2019
What you know how to do (at this point in the course)

Position objects and the camera in the world

Determine the position of objects relative to the camera

Project objects onto the screen

Sample triangle coverage

Compute triangle attribute values at covered sample points (Color, texture coords, depth)

Sample texture maps
Texture mapping review
Per-vertex information

- Inputs:
  - Per-vertex position \([x,y,z]\)
  - Per-vertex texture coordinates \([u,v]\)

\( (u=0.4, v=0.7) \)

Defines mapping from domain of surface, to domain of texture map
Linearly interpolate texture coordinate samples

\( b - a \) and \( c - a \) form a non-orthogonal basis for points in triangle (origin at \( a \))

\[
x = a + \beta(b - a) + \gamma(c - a)
= (1 - \beta - \gamma)a + \beta b + \gamma c
= \alpha a + \beta b + \gamma c
\]

\( \alpha + \beta + \gamma = 1 \)

UV at \( x \) is linear combination of UV at three triangle vertices.

\[
x_{uv} = \alpha a_{uv} + \beta b_{uv} + \gamma c_{uv}
\]
Barycentric coordinates as ratio of areas

Given XYZ positions of triangle vertices, compute barycentric coordinates…
Interpolating texture coordinates in 2D

- But consider assignment 1…

- You are given 2D position of triangle coordinates, and you have to sample coverage (and now UV) at a given 2D screen point (X,Y)
Perspective incorrect interpolation

Due to perspective projection (homogeneous divide), barycentric interpolation of values on a triangle with vertices of different depths is not linear in screen XY coordinates.

Attribute values must be interpolated linearly in 3D space.
Perspective-correct interpolation

Assume triangle attribute varies linearly across the triangle

Attribute’s value at 3D (non-homogeneous) point \( P = [x \ y \ z]^T \) is:

\[
f(x, y, z) = ax + by + cz
\]

Project \( P \), get 2D homogeneous representation:

\[
[x_{2D-H} \ y_{2D-H} \ w]^T = [x \ y \ z]^T
\]

Rewrite attribute equation for \( f \) in terms of 2D homogeneous coordinates:

\[
f = ax_{2D-H} + by_{2D-H} + cw
\]

\[
\frac{f}{w} = a \frac{x_{2D-H}}{w} + b \frac{y_{2D-H}}{w} + c
\]

\[
\frac{f}{w} = ax_{2D} + by_{2D} + c
\]

Where \( [x_{2D} \ y_{2D}]^T \) are projected screen 2D coordinates (after homogeneous divide)

So \( \frac{f}{w} \) is affine function of 2D screen coordinates:

\[
[x_{2D} \ y_{2D}]^T
\]
Direct evaluation of surface attributes

For any surface attribute (with value defined at triangle vertices as: $f_a, f_b, f_c$)

$w$ coordinate of vertex $a$ after perspective projection transform

\[
\frac{f_a}{w} = A a_x + B a_y + C
\]

\[
\frac{f_b}{w} = A b_x + B b_y + C
\]

\[
\frac{f_c}{w} = A c_x + B c_y + C
\]

3 equations, solve for 3 unknowns ($A, B, C$)

This is done as a per triangle “setup” computation prior to sampling, just like you computed edge equations for evaluating coverage.
Efficient perspective-correct interpolation

Attribute values vary linearly across triangle in 3D, but not in projected screen XY
Projected attribute values \( (f/w) \) are affine functions of screen XY!

To evaluate surface attribute \( f \) at every covered sample:

Evaluate \( 1/w (x,y) \) (from precomputed equation for value \( 1/w \))

Reciprocate \( 1/w (x,y) \) to get \( w(x,y) \)

For each triangle attribute:

Evaluate \( f/w (x,y) \) (from precomputed equation for value \( f/w \))

Multiply \( f/w (x,y) \) by \( w(x,y) \) to get \( f(x,y) \)

Works for any surface attribute \( f \) that varies linearly across triangle:

e.g., color, depth, texture coordinates
What else do you need to know to render a picture like this?

Surface representation
How to represent complex surfaces?

Occlusion
Determining which surface is visible to the camera at each sample point

Lighting/materials
Describing lights in scene and how materials reflect light.
Course roadmap: what’s coming...

Drawing Things

Key concepts:
- Sampling (and anti-aliasing)
- Coordinate Spaces and Transforms
- Rasterization and texturing via sampling

Geometry Processing

Materials and Lighting

Midterm

Introduction

Drawing a triangle (by sampling)

Transforms and coordinate spaces

Perspective projection and texture sampling

Today: putting it all together: end-to-end rasterization pipeline
Occlusion using the Depth Buffer
Occlusion: which triangle is visible at each covered sample point?

Opaque Triangles

50% transparent triangles
Depth buffer (aka “Z buffer”)

Color buffer:
(stores color per sample... e.g., RGB)

Depth buffer:
(stores depth per sample)

Stores depth of closest surface drawn so far
black = close depth
white = far depth
Depth buffer (a better look)

Color buffer (stores color measurement per sample, eg., RGB value per sample)
Depth buffer (a better look)

Corresponding depth buffer after rendering all triangles (stores closest scene depth per sample)
Occlusion using the depth-buffer ("Z-buffer")

For each coverage sample point, the depth-buffer stores depth of closest triangle at this sample point that has been processed by the renderer so far.

Closest triangle at sample point \((x,y)\) is triangle with minimum depth at \((x,y)\)

Initial state of depth buffer before rendering any triangles (all samples store farthest distance)

Grayscale value of sample point used to indicate distance

Black = small distance
White = large distance
Assume we have a triangle defined by the screen-space 2D position and distance ("depth") from the camera of each vertex.

\[
\begin{align*}
[ p_{0x} & p_{0y} ]^T, & \quad d_0 \\
[ p_{1x} & p_{1y} ]^T, & \quad d_1 \\
[ p_{2x} & p_{2y} ]^T, & \quad d_2 
\end{align*}
\]

How do we compute the depth of the triangle at covered sample point \((x, y)\)?

Interpolate it just like any other attribute that varies linearly over the surface of the triangle.
Example: rendering three opaque triangles
Occlusion using the depth-buffer (Z-buffer)

Processing yellow triangle:
depth = 0.5

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = samples that pass depth test

Color buffer contents

Depth buffer contents
## Occlusion using the depth-buffer (Z-buffer)

### After processing yellow triangle:

<table>
<thead>
<tr>
<th>Color buffer contents</th>
<th>Depth buffer contents</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Color buffer contents" /></td>
<td><img src="image2" alt="Depth buffer contents" /></td>
</tr>
</tbody>
</table>

- Grayscale value of sample point used to indicate distance.
  - White = large distance
  - Black = small distance
  - Red = samples that pass depth test
Occlusion using the depth-buffer (Z-buffer)

Processing blue triangle:
depth = 0.75

Depth buffer contents

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = samples that pass depth test

Color buffer contents
Occlusion using the depth-buffer (Z-buffer)

After processing blue triangle:

<table>
<thead>
<tr>
<th>Color buffer contents</th>
<th>Depth buffer contents</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Color buffer" /></td>
<td><img src="image2" alt="Depth buffer" /></td>
</tr>
</tbody>
</table>

Grayscale value of sample point used to indicate distance:
- White = large distance
- Black = small distance
- Red = samples that pass depth test
Occlusion using the depth-buffer (Z-buffer)

Processing red triangle:
depth = 0.25

Color buffer contents

Depth buffer contents

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = samples that pass depth test
Occlusion using the depth-buffer (Z-buffer)

After processing red triangle:

Color buffer contents

Depth buffer contents

Grayscale value of sample point used to indicate distance
White = large distance
Black = small distance
Red = samples that pass depth test
Occlusion using the depth buffer (opaque surfaces)

```cpp
bool pass_depth_test(d1, d2) {
    return d1 < d2;
}

depth_test(tri_d, tri_color, x, y) {
    if (pass_depth_test(tri_d, depth_buffer[x][y]) {
        // triangle is closest object seen so far at this
        // sample point. Update depth and color buffers.
        depth_buffer[x][y] = tri_d;   // update depth_buffer
        color[x][y] = tri_color;      // update color buffer
    }
}
```
Does depth-buffer algorithm handle interpenetrating surfaces?

Of course!

Occlusion test is based on depth of triangles at a given sample point. The relative depth of triangles may be different at different sample points.
Does depth-buffer algorithm handle interpenetrating surfaces?

Of course!

Occlusion test is based on depth of triangles *at a given sample point*. The relative depth of triangles may be different at different sample points.
Does depth buffer work with super sampling?

Of course! Occlusion test is per sample, not per pixel!

This example: green triangle occludes yellow triangle
Color buffer contents
Color buffer contents (4 samples per pixel)
Final resampled result

Note anti-aliasing of edge due to filtering of green and yellow samples.
Summary: occlusion using a depth buffer

- Store one depth value per coverage sample (not per pixel!)
- Constant space per sample
  - Implication: constant space for depth buffer
- Constant time occlusion test per covered sample
  - Read-modify write of depth buffer if “pass” depth test
  - Just a depth buffer read if “fail”
- Not specific to triangles: only requires that surface depth can be evaluated at a screen sample point

But what about semi-transparent surfaces?
Compositing
Representing opacity as alpha

Alpha describes the opacity of an object
- Fully opaque surface: $\alpha = 1$
- 50% transparent surface: $\alpha = 0.5$
- Fully transparent surface: $\alpha = 0$

Red triangle with decreasing opacity

$\alpha = 1$  $\alpha = 0.75$  $\alpha = 0.5$  $\alpha = 0.25$  $\alpha = 0$
Alpha: coverage analogy

- Can think of alpha as describing the opacity of a semi-transparent surface
- Or... as partial coverage by fully opaque object
  - consider a screen door

\[ \alpha = 0.5 \]

(Squint at this slide and the scene on the left and the right will appear similar)
Alpha: additional channel of image (rgba)

\[ \alpha \text{ of foreground object} \]
Over operator:

Composite image $B$ with opacity $\alpha_B$ over image $A$ with opacity $\alpha_A$

$B \over A$  

$A \over B$

$A \over B \neq B \over A$

“Over” is not commutative

Koala over NYC
Over operator: non-premultiplied alpha

Composite image $B$ with opacity $\alpha_B$ over image $A$ with opacity $\alpha_A$

First attempt: (represent colors as 3-vectors, alpha separately)

\[
A = \begin{bmatrix} A_r & A_g & A_b \end{bmatrix}^T
\]
\[
B = \begin{bmatrix} B_r & B_g & B_b \end{bmatrix}^T
\]

Composited color:

\[
C = \alpha_B B + (1 - \alpha_B) \alpha_A A
\]

Appearance of semi-transparent $A$

Appearance of semi-transparent $B$

What $B$ lets through

Composite alpha:

\[
\alpha_C = \alpha_B + (1 - \alpha_B) \alpha_A
\]

"Over" is not commutative
Premultiplied alpha

- Represent (potentially transparent) color as a 4-vector where RGB values have been premultiplied by alpha

\[ A' = [\alpha_A A_r \quad \alpha_A A_g \quad \alpha_A A_b \quad \alpha_A]^T \]

Example: 50% opaque red
[0.5, 0.0, 0.0, 0.5]

Example: 75% opaque magenta
[0.75, 0.0, 0.75, 0.75]
Over operator: using premultiplied alpha

Composite image B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

Non-premultiplied alpha representation:

\[
A = \begin{bmatrix} A_r & A_g & A_b \end{bmatrix}^T \\
B = \begin{bmatrix} B_r & B_g & B_b \end{bmatrix}^T \\
C = \alpha_B B + (1 - \alpha_B) \alpha_A A
\]

two multiplies, one add

(Referring to vector ops on colors)

Composite alpha:

\[
\alpha_C = \alpha_B + (1 - \alpha_B) \alpha_A
\]

Premultiplied alpha representation:

\[
A' = \begin{bmatrix} \alpha_A A_r & \alpha_A A_g & \alpha_A A_b & \alpha_A \end{bmatrix}^T \\
B' = \begin{bmatrix} \alpha_B B_r & \alpha_B B_g & \alpha_B B_b & \alpha_B \end{bmatrix}^T \\
C' = B + (1 - \alpha_B) A
\]

\[
B \text{ over } A
\]
Fringing

Poor treatment of color/alpha can yield dark “fringing”:

foreground color

foreground alpha

background color

fringing

no fringing
No fringing
Fringing (…why does this happen?)
A problem with non-premultiplied alpha

- Suppose we upsample an image w/ an alpha mask, then composite it onto a background.
- How should we compute the interpolated color/alpha values?
- If we interpolate color and alpha separately, then blend using the non-premultiplied “over” operator, here’s what happens:

Notice black “fringe” that occurs because we’re blending, e.g., 50% blue pixels using 50% alpha, rather than, say, 100% blue pixels with 50% alpha.
Eliminating fringe w/ premultiplied “over”

If we instead use the premultiplied “over” operation, we get the correct alpha:

\[
\text{upsampled color} + (1-\alpha) \times \text{background} = \text{composite image w/ no fringe}
\]
Another problem with non-premultiplied alpha

Consider pre-filtering a texture with an alpha matte

![Image of a leaf with alpha channel]

Downsampling non-premultiplied alpha image results in 50% opaque brown.

\[
0.25 \times ((0, 1, 0, 1) + (0, 1, 0, 1) + (0, 0, 0, 0) + (0, 0, 0, 0)) = (0, 0.5, 0, 0.5)
\]

Result of filtering premultiplied image

Desired filtered result composed over white
Common use of textures with alpha: foliage

[Image credit: SpeedTree Cinema 8]
Foliage example

[Image credit: SpeedTree Cinema 8]
Another problem: applying “over” repeatedly

Consider composite image C with opacity $\alpha_C$ over B with opacity $\alpha_B$ over image A with opacity $\alpha_A$

$$A = \begin{bmatrix} A_r & A_g & A_b \end{bmatrix}^T$$
$$B = \begin{bmatrix} B_r & B_g & B_b \end{bmatrix}^T$$
$$C = \alpha_B B + (1 - \alpha_B) \alpha_A A$$
$$\alpha_C = \alpha_B + (1 - \alpha_B) \alpha_A$$

Consider first step of of compositing 50% red over 50% red:

$$C = \begin{bmatrix} 0.75 & 0 & 0 \end{bmatrix}^T$$
$$\alpha_C = 0.75$$

Wait... this result is the premultiplied color!

So “over” for non-premultiplied alpha takes non-premultiplied colors to premultiplied colors (“over” operation is not closed)

Cannot compose “over” operations on non-premultiplied values: 

over(C, over(B, A))

There is a closed form for non-premultiplied alpha:

$$C = \frac{1}{\alpha_C} (\alpha_B B + (1 - \alpha_B) \alpha_A A)$$
Summary: advantages of premultiplied alpha

- Simple: compositing operation treats all channels (rgb and a) the same
- Closed under composition
- Better representation for filtering textures with alpha channel
- More efficient than non-premultiplied representation: “over” requires fewer math ops
Color buffer update: semi-transparent surfaces

Assume: color buffer values and tri_color are represented with premultiplied alpha

```cpp
over(c1, c2) {
    return c1 + (1-c1.a) * c2;
}

update_color_buffer(tri_z, tri_color, x, y) {
    // Note: no depth check, no depth buffer update
    color[x][y] = over(tri_color, color[x][y]);
}
```

What is the assumption made by this implementation?
**Triangles must be rendered in back to front order!**

What if triangles are rendered in front to back order?
**Modify code: over(color[x][y], tri_color)**
Putting it all together *

Consider rendering a mixture of opaque and transparent triangles

Step 1: render opaque surfaces using depth-buffered occlusion
   If pass depth test, triangle overwrites value in color buffer at sample

Step 2: disable depth buffer update, render semi-transparent surfaces in back-to-front order.
   If pass depth test, triangle is composited OVER contents of color buffer at sample

* If this seems a little complicated, you will enjoy the simplicity of using ray tracing algorithm for rendering. More on this later in the course, and in CS348B
Combining opaque and semi-transparent triangles

Assume: color buffer values and tri_color are represented with premultiplied alpha

// phase 1: render opaque surfaces
update_color_buffer(tri_z, tri_color, x, y) {
   if (pass_depth_test(tri_z, zbuffer[x][y]) {
      color[x][y] = tri_color;
      zbuffer[x][y] = tri_z;
   }
}

// phase 2: render semi-transparent surfaces
update_color_buffer(tri_z, tri_color, x, y) {
   if (pass_depth_test(tri_z, zbuffer[x][y]) {
      // Note: no depth buffer update
      color[x][y] = over(tri_color, color[x][y]);
   }
}
End-to-end rasterization pipeline
(“real-time graphics pipeline”)
Command: draw these triangles!

Inputs:

list_of_positions = {
    v0x, v0y, v0z,
    v1x, v1y, v1x,
    v2x, v2y, v2z,
    v3x, v3y, v3x,
    v4x, v4y, v4z,
    v5x, v5y, v5x
};

list_of_texcoords = {
    v0u, v0v,
    v1u, v1v,
    v2u, v2v,
    v3u, v3v,
    v4u, v4v,
    v5u, v5v
};

Object-to-camera-space transform: \( T \)

Perspective projection transform \( P \)

Size of output image \( (W, H) \)

Use depth test / update depth buffer: YES!
Step 1:
Transform triangle vertices into camera space
(apply modeling and camera transform)
Step 2:
Apply perspective projection transform to transform triangle vertices into normalized coordinate space

Camera-space positions: 3D

Normalized space positions

Note: I’m illustrating normalized 3D space after the homogeneous divide, it is more accurate to think of this volume in 3D-H space as defined by: 
(-w, -w, -w, w) and (w, w, w, w)
Step 3: clipping

- Discard triangles that lie complete outside the unit cube (culling)
  - They are off screen, don’t bother processing them further
- Clip triangles that extend beyond the unit cube to the cube
  - Note: clipping may create more triangles
Step 4: transform to screen coordinates

Transform vertex xy positions from normalized coordinates into screen coordinates (based on screen w,h)
Step 5: setup triangle (triangle preprocessing)

Compute triangle edge equations
Compute triangle attribute equations

\[
\begin{align*}
E_{01}(x, y) & \quad U(x, y) \\
E_{12}(x, y) & \quad V(x, y) \\
E_{20}(x, y) \\
\frac{1}{w}(x, y) \\
Z(x, y)
\end{align*}
\]
Step 6: sample coverage

Evaluate attributes $z, u, v$ at all covered samples
Step 6: compute triangle color at sample point

e.g., sample texture map *

* So far, we’ve only described computing triangle’s color at a point by interpolating per-vertex colors, or by sampling a texture map. Later in the course, we’ll discuss more advanced algorithms for computing its color based on material properties and scene lighting conditions.
Step 7: perform depth test (if enabled)

Also update depth value at covered samples (if necessary)
Step 8: update color buffer (if depth test passed)
Step 9:

- Repeat steps 1-8 for all triangles in the scene!
Real time graphics APIs

- OpenGL
- Microsoft Direct3D
- Apple Metal

- You now know a lot about the algorithms implemented underneath these APIs: drawing 3D triangles (key transformations and rasterization), texture mapping, anti-aliasing via supersampling, etc.

- Internet is full of useful tutorials on how to program using these APIs
OpenGL/Direct3D graphics pipeline

Structures rendering computation as a series of operations on vertices, primitives, fragments, and screen samples

Operations on vertices
- Vertex Processing
  - Vertex stream

Operations on primitives (triangles, lines, etc.)
- Primitive Processing
  - Primitive stream

Operations on fragments
- Fragment Generation (Rasterization)
  - Fragment stream

Operations on screen samples
- Screen sample operations (depth and color)
  - Shaded fragment stream

Input: vertices in 3D space

Vertices in positioned in normalized coordinate space

Triangles positioned on screen

Fragments (one fragment per covered sample)

Shaded fragments

Output: image (pixels)

* Several stages of the modern OpenGL pipeline are omitted
OpenGL/Direct3D graphics pipeline

Operations on vertices
- Vertex stream
- Vertices in 3D space
- Parameters needed to compute position on vertices in normalized coordinates (e.g., transform matrices)
- "Shader" programs that define behavior of vertex and fragment stages

Operations on primitives (triangles, lines, etc.)
- Primitive stream
- Parameters needed to compute color of fragments (e.g., textures)

Operations on fragments
- Fragment stream
- Shaded fragment stream
- Input vertices in 3D space
- Content not specified

Operations on screen samples
- Screen sample operations (depth and color)
- Screen sample operations not specified

Pipeline inputs:
- Input vertex data
- Parameters needed to compute position on vertices in normalized coordinates (e.g., transform matrices)
- Parameters needed to compute color of fragments (e.g., textures)
- "Shader" programs that define behavior of vertex and fragment stages

* several stages of the modern OpenGL pipeline are omitted
Shader programs

Define behavior of vertex processing and fragment processing stages
Describe operation on a single vertex (or single fragment)

Example GLSL fragment shader program

```glsl
uniform sampler2D myTexture;
uniform vec3 lightDir;
varying vec2 uv;
varying vec3 norm;

void diffuseShader()
{
    vec3 kd;
    kd = texture2d(myTexture, uv);
    kd *= clamp(dot(-lightDir, norm), 0.0, 1.0);
    gl_FragColor = vec4(kd, 1.0);
}
```

Shader function executes once per fragment.

Outputs color of surface at sample point corresponding to fragment.
(this shader performs a texture lookup to obtain the surface's material color at this point, then performs a simple lighting computation)

Program parameters

Per-fragment attributes (interpolated by rasterizer)

Sample surface albedo (reflectance color) from texture

Shader outputs surface color

Modulate surface albedo by incident irradiance (incoming light)
Texture coordinate visualization
Defines mapping from point on surface to point \((uv)\) in texture domain

Red channel = \(u\), Green channel = \(v\)
So \(uv=(0,0)\) is black, \(uv=(1,1)\) is yellow
Rendered result
Goal: render very high complexity 3D scenes

- hundreds of thousands to millions of triangles in a scene
- Complex vertex and fragment shader computations
- High resolution screen outputs (2-4 Mpixel + supersampling)
- 30-60 fps
Graphics pipeline implementation: GPUs

Specialized processors for executing graphics pipeline computations

Discrete GPU card
(NVIDIA GeForce Titan X)

Integrated GPU: part of modern Intel CPU chip
Modern GPUs offer ~2-4 TFLOPs of performance for executing vertex and fragment shader programs.

T-OP's of fixed-function compute capability over here

Take Kayvon’s Visual Computing Systems course (CS348V) for more details!
Summary

- Occlusion resolved independently at each screen sample using the depth buffer
- Alpha compositing for semi-transparent surfaces
  - Premultiplied alpha forms simply repeated composition
  - “Over” compositing operations is not commutative: requires triangles to be processed in back-to-front (or front-to-back) order
- Graphics pipeline:
  - Structures rendering computation as a sequence of operations performed on vertices, primitives (e.g., triangles), fragments, and screen samples
  - Behavior of parts of the pipeline is application-defined using shader programs.
  - Pipeline operations implemented by highly, optimized parallel processors and fixed-function hardware (GPUs)